Some Studies of Ground Water Flow through Highly Permeable Soils

by

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Introduction

WATER for irrigation, human consumption, industrial and other purposes is often obtained from wells which are either dug or bored into the underlying ground. The yield from such wells, among other things, depends on the type and characteristics of the soil through which the water moves to reach the wells. In most cases the soil is fine enough that the speed of movement of water is extremely small being of the order of a few metres a day. The laws governing such a type of flow, called laminar flow, have been exhaustively studied previously by many investigators.

The famous Darcy's law, which states

$$v = ki$$

where v = mean velocity of flow, i = rate of loss of head with distance and k = coefficient of permeability, was evolved in 1856. In view of the fundamental importance of Darcy's law, a number of investigators (1962) like Dupuit, Nutting, Hazen, Kozeny, Carman and Fancher investigated the physical significance of the Darcy's coefficient of permeability and the range of validity of Darcy's law. Hubbert (1956) solved the Navier-Stokes equations and presented a theoretical background to the form of Darcy's law.

Very often the yield from a well is determined on basis of the coefficient of permeability determined from pumping tests using the Thiem formula which is derived on the assumption of the validity of Darcy's law. If a well receives water from a region composed of highly coarse grained soils with comparatively large sizes of pores between soil particles, estimation of the yield based on the Thiem formula may lead to serious errors. Anandakrishnan and Varadarajulu (1963), Ward (1964), Dudgeon (1966) and Wright (1968) are some of the investigators that conducted studies on the non-linear flow through granular media. Essentially three forms of flow relationships were suggested. They are

(1) $i = k' v^n$ (Anandakrishnan and Varadarajulu and Dudgeon),

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(2) $i = a\,v + b\,v^2$ (Ward), and

(3) Friction factor—Reynolds' Number Correlations.

Recent studies at the Andhra University, Waltair [Subba Rao (1969)] were directed towards obtaining the velocity-hydraulic gradient relationship for flow of water through highly permeable soils represented by uniformly packed and fully saturated beds of gravel. The present paper describes some of these results including the effects of porosity, grain-size and tortuosity on the flow relationship describing non-Darcian flow.

Experimental Apparatus

The experimental set-up (See Figure 1) consisted of a 0·3 m × 0·3 m × 3 m long M. S. recirculating-*cum*-storage tank from which water is pumped by a 3 H.P. centrifugal pump through a 1·5 m long, 7·6 cm nominal diameter G.I. pipe which was filled with the material to be tested. Four piezometers of 1·6 mm size were provided at 15·2 cm spacing to measure the loss of head. A 0·6 m long approach section and a 0·45 m long discharge section were available on either side of the test section. The water passing through the permeameter was collected in a 0·3 m × 0·3 m × 0·6 m M.S. collecting tank and was subsequently returned to the storage tank. Plastic tubes of equal length were led from the top three piezometer tappings to a manifold which was connected to a mercury-water U-tube manometer. The bottom piezometer tapping was connected directly to the manometer. The material inside the permeameter was supported at the bottom and contained at the top by perforated M. S. plates. The permeameter was kept in a vertical position by means of a frame erected for the purpose. For all the tests the direction of flow was upwards.

Experimental Procedure

Hand-picked, fairly round river gravel was procured and sieved into various sizes using 6·3 mm, I.S.480, I.S. 340, I.S.240, I.S.120 and I.S.85 sieves. Ordinary tap water was used in all the tests.

Each gravel was placed in the permeameter by freely pouring it from the top of the permeameter. The amount of gravel putin was preweighed and the length of the gravel bed was noted before the permeameter was closed with a dummy after placing the porous cover plate. To obtain different porosities, the material was manually compacted with a plunger by giving 10 to 20 blows with a uniform drop of 10·2 to 15·2 cm. The procedure followed was chosen randomly to get a fairly uniform porosity throughout the length of the gravel bed. Around 12 kg of gravel of each size was used every time in the permeameter. Bleeding of the air from the apparatus, from the tubings, from the manifold and from the manometer was done by operating the relevant cocks. Discharge through the gravel bed was varied by operating the inlet value. At every flow rate the manometer readings for all the three sets of piezometers were noted and the mean value of the hydraulic gradient computed. The discharge was measured by volumetric method. The geometric mean of the mesh sizes of the two consecutive sieves, pertaining to each gravel, was taken as the mean grain-size for calculating the Reynolds' number and the Fanning friction factor. Specific gravity of the

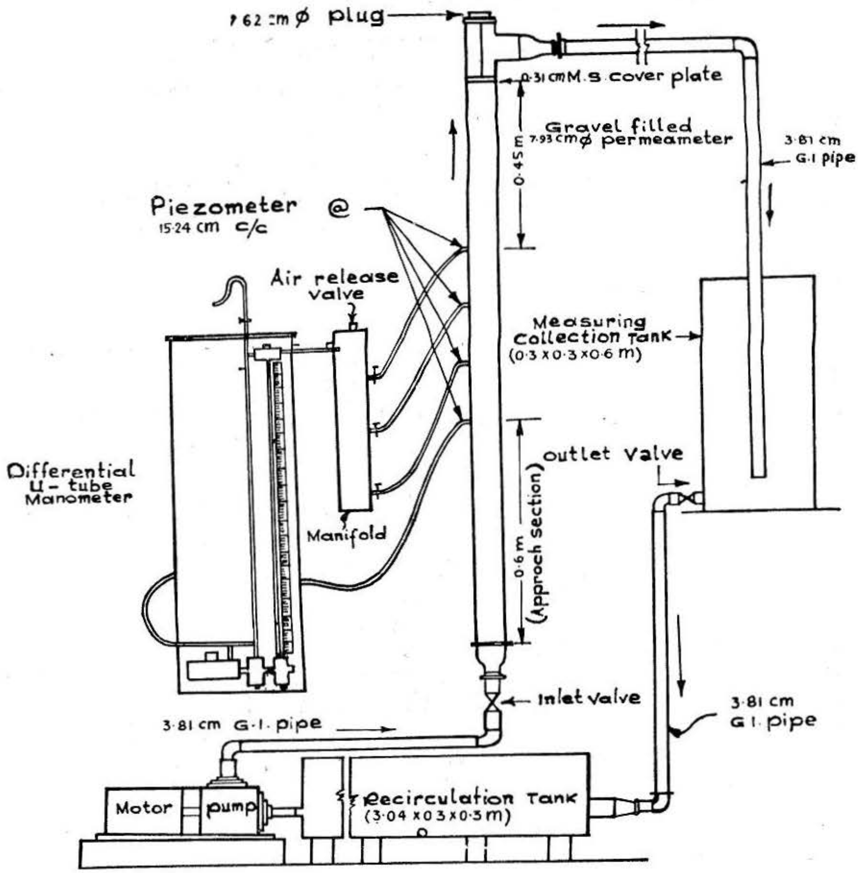


FIGURE 1 : Schematic diagram of the equipment for tests with small size gravel.

gravels was determined by using a pyknometer. The temperatures of water before and after each test were recorded.

Discussion of Results

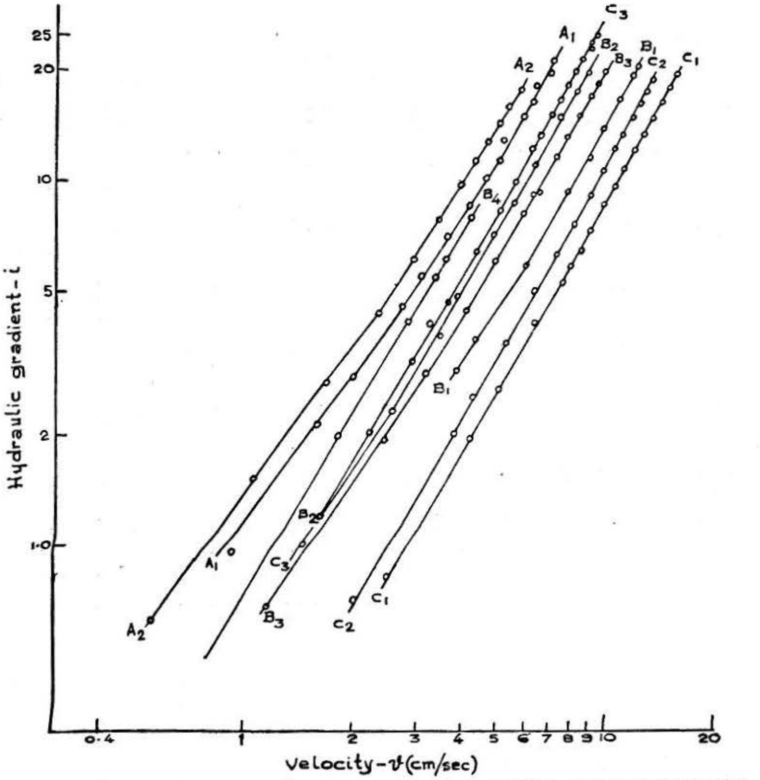
VELOCITY-HYDRAULIC GRADIENT RELATIONSHIP

Figures 2 and 3 show the log-log plots of the hydraulic gradient *versus* average velocity data for the 15 test runs made with five different sizes of the media. As can be seen, each particular medium was tested at more than one porosity value. All the plots show that the hydraulic gradient-velocity relationship appears to be of a discontinuous form of the type $i=k' v^n$ where i=hydraulic gradient, v=average velocity, k' and n are coefficients. Excepting for the media B_4 and D_3, two or three flow regimes plotting as straight lines on the log-log plot were observed for all other media. The values of k' and n, as determined from these log-log plots, for all the media and for all the flow regimes are given in Table I. In the last column of this table are shown the ranges of hydraulic gradient values governing each flow regime.

Table I shows that the values of n were between 1·34 and 2·25. This shows that all the tests were exclusively conducted in post-linear regimes of flow. It is, however, improbable that the values of n can be larger than 2 as in the cases of media E_1 and E_2 (d=0·55 cm) which show n values larger than 2. As in the case of flow through circular pipes, the fully turbulent flow condition can be expected to have a value of the exponent equal to 2.

FRICTION FACTOR—REYNOLDS' NUMBER RELATIONSHIP

Figure 4 shows the log plot of Fanning friction factor *versus* Reynolds' number for all the tests. The geometric mean of the mesh sizes of the gravel medium was taken as the characteristic diameter in both the quantities. A wide range of Reynolds' numbers from 7·5 to about 1600 was covered in the tests. As can be seen, the data of each material trace a smooth curve falling down with increased Reynolds' numbers registering a very nearly constant value of friction factor at



Material	Mean grain size	porosity
A1	0·101 cm.	39·99 %.
A2	0·101 "	38·06 "
B1	0·1697 "	43·57 "
B2	0·1697 "	41·57
B3	0·1697 "	40·32
B4	0·1697 "	39·16%
C1	0·2857 "	43·04
C2	0·2857 "	42·29
C3	0·2857 "	40·25

FIGURE 2 : Hydraulic gradient versus velocity (for different sizes at different orosities).

TABLE I

Values of k' and n.

Test No. (1)	Material (2)	Porosity (%) (3)	Geometric mean size (cm) (4)	n (5)	k' (6)	Range of hydraulic gradient (7)
1.	A_1 : I.S. 120—I.S. 85	39·99	0·10	1·34 1·54 1·96	1·17 0·97 0·38	0·90— 3·70 3·70—12·50 12·50—21·00
2.	A_2 : —do—	38·06	0·10	1·36 1·57	1·39 1·15	0·60— 4·20 4·20—15·60
3.	B_1 : I.S. 240—I.S. 120	43·57	0·17	1·50 1·79	0·39 0·29	2·90— 5·60 5·60—20·00
4.	B_2 : —do—	41·57	0·17	1·48 1·75	0·59 0·59	1·20— 2·80 2·80—20·00
5.	B_3 : —do—	40·32	0·17	1·46 1·72	0·56 0·39	0·70— 3·40 3·40—20·00
6.	B_4 : —do—	39·16	0·17	1·68	0·71	0·50— 8·00
7.	C_1 : I.S. 340—I.S. 240	43·04	0·29	1·70 1·83	0·17 0·13	0·80 — 4·00 4·00—19·00
8.	C_2 : —do—	42·29	0·29	1·70 1·84	0·21 0·16	0·70— 5·00 5·00—18·00
9.	C_3 : —do—	40·25	0·29	1·66 1·84	0·54 0·42	1·00—6·00 6·00—25·00
10.	D_1 : I.S. 480—I.S. 340	38·44	0·40	1·75 1·86	0·11 0·09	0·90— 4·80 4·80—19·00
11.	D_2 : —do—	36·69	0·40	1·75 1·88	0·15 0·12	0·80— 2·70 2·70—18·00
12.	D_3 : —do—	35·24	0·40	1·83	0·21	0·90—13·00
13.	E_1 : 6·3 mm—I.S. 480	37·18	0·55	1·77 1·90 2·25	0·08 0·06 0·02	0·30— 4·00 4·00—14·00 14·00—18·00
14.	E_2 : —do—	35·64	0·55	1·84 2·03	0·08 0·05	0·80—11·50 11·50—18·00
15.	E_3 : —do—	34·58	0·55	1·88 1·98	0·12 0·09	0·80—13·50 13·50—20·00

higher Reynolds' numbers. This is as it should be since with increasing inertial effects as represented by higher Reynolds' numbers the flow should be independent of viscous effects as evidenced by similar plots for flow through circular pipes. However, a generalised friction factor—Reynolds' number plot, representing all sizes, does not seem to be feasible in view of various other factors like porosity, size distribution, particle shape, particle roughness, tortuosity, *etc.*, which can definitely be expected to have some influence on the flow phenomenon. In the

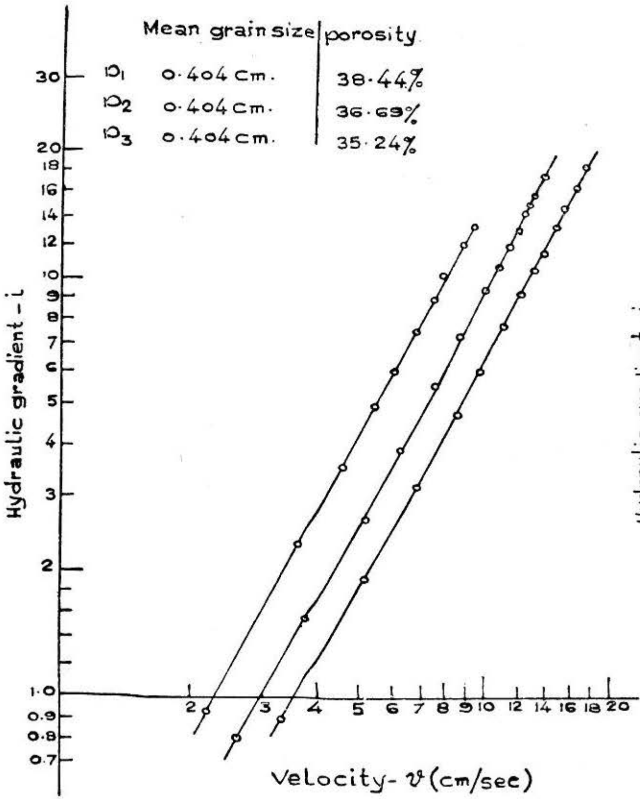


Figure 3(a):- Hydraulic gradient Vs velocity

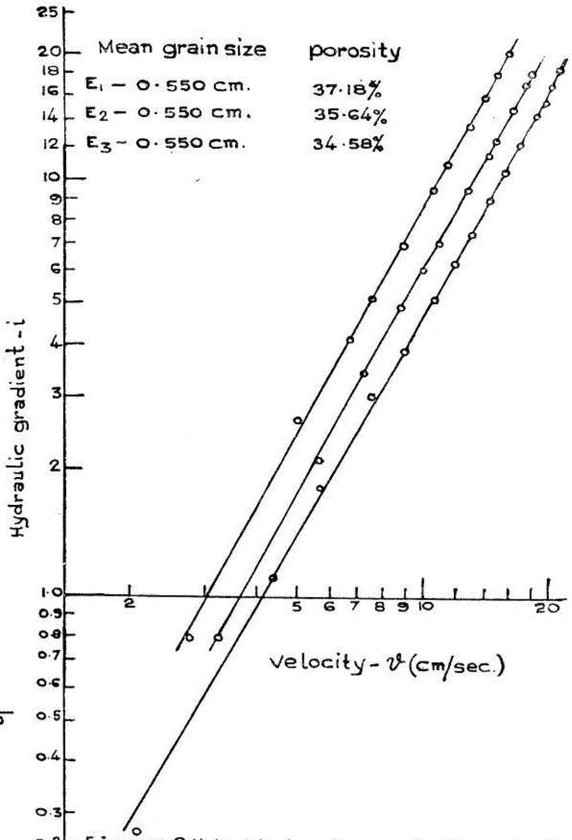


Figure 3(b):- Hydraulic gradient Vs velocity

FIGURE 3 (a & b) : Hydraulic gradient versus velocity.

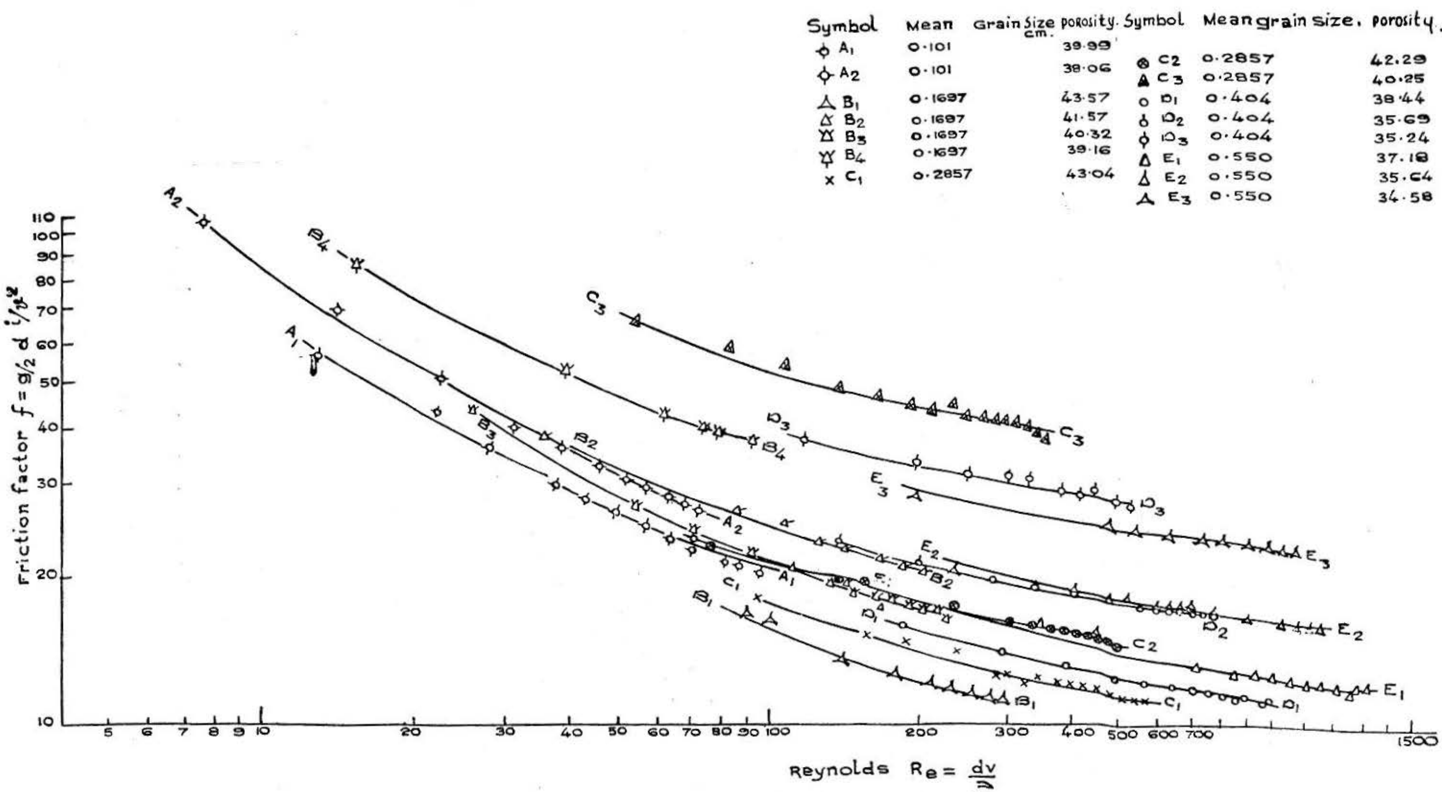


FIGURE 4 : Friction factor—Reynolds' number.

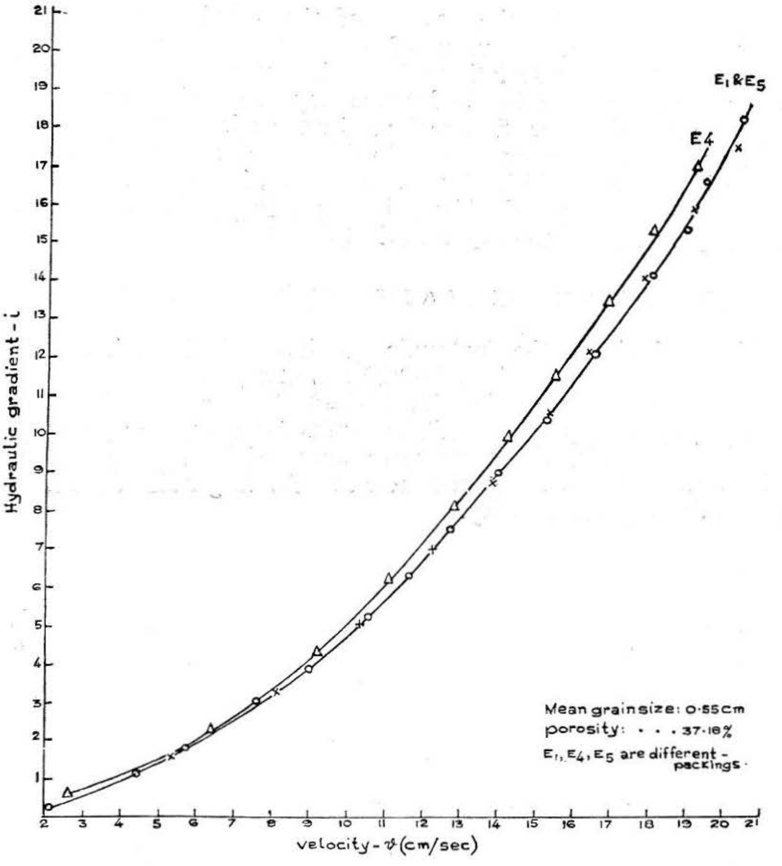


FIGURE 5 : Hydraulic gradient-velocity for different tortuosities.

absence of a generalised $f-R_e$ plot, it may be better to express the results in terms of velocity-hydraulic gradient relationships.

EFFECT OF POROSITY ON THE FLOW REGIME

The tests on gravel A ($d=0{\cdot}10$ cm) were conducted at two different porosities, those on B ($d=0{\cdot}17$ cm) at four porosities. Gravels C, D, and E were tested at three different porosities each. As can be seen from Table I, the values of k' and n were different for a given material at different porosities, at times the number of flow regimes changing as a consequence of change in porosity as in the cases of media A_1 and A_2, B_3 and B_4, D_2 and D_3 and E_1 and E_2.

EFFECT OF TORTUOSITY OF A MEDIUM ON THE FLOW REGIME

Three tests were conducted on gravel E ($d=0{\cdot}55$ cm) at a constant porosity of 37·18 percent. The permeameter was emptied and filled up with the same quantity of material occupying the same volume so that the porosity did not change. By this procedure, the relative disposition of the porechannels is expected to change with a corresponding change in the tortuosity factor. The results of these tests are plotted in Figure 5. As seen from the figure, the results of two tests completely coincided, the results of the third deviating slightly from the other two. In view of cent percent agreement of the results of two tests, the deviation of the third might be due to experimental errors. Otherwise a band of curves may be obtained to describe the limits within which the flow regime, for a particular size and porosity, would be defined. Some more tests in this regard are required before a definite conclusion could be drawn. However, it may be stated that the effect of tortuosity is not significant in large size gravels as they do not contain capillaries and micropores (1962).

EFFECT OF GRAIN-SIZE OF A MEDIUM ON THE FLOW REGIME

Figure 6 shows the hydraulic gradient-velocity relationship of gravels, B (d=0·17 cm) and C (d=0·29 cm) at a porosity of 42·3 percent. Figure 7 shows the results of media D (d=0·40 cm) and E (d=0·55 cm) at a porosity of 37·18 percent. Where results for a common porosity were not readily available, the same were obtained from interpolation. As can be anticipated, the graphs show that for larger sizes, the head loss is smaller for a given velocity.

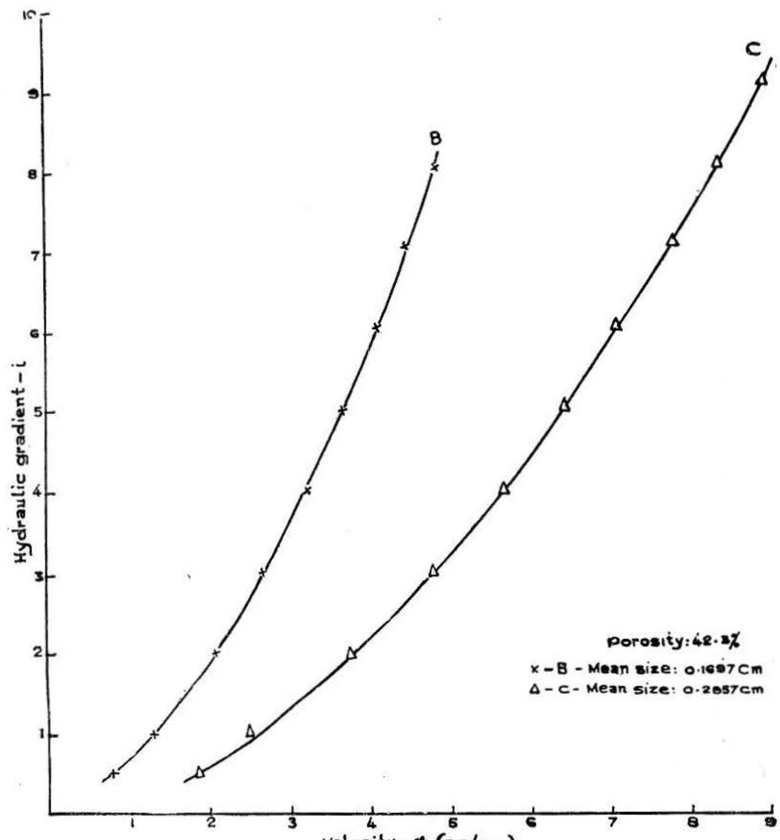


FIGURE 6: Hydraulic gradient-velocity (for different sizes at the same porosity).

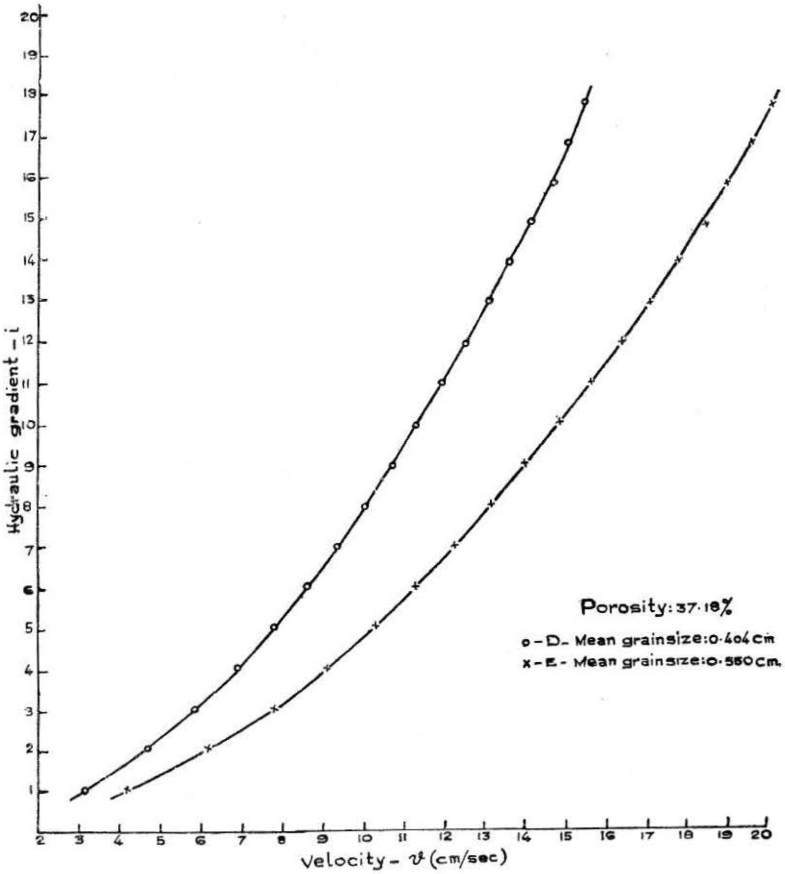


FIGURE 7 : **Hydraulic gradient-velocity (for different sizes at the same porosity).**

Conclusions

The hydraulic gradient-velocity relationship for flow of water through gravel is non-linear and an exponential relationship like $i=k' v^n$ describes the flow satisfactorily. For a given medium, various non-linear flow regimes are possible with different values n and k' describing the various regimes. As the size of the medium increases, value of n increases and tends to approach 2. As the porosity increases for a given medium the value of n increases. For a medium at a particular porosity, the changes in the regime are abrupt, the value of n increasing, from one regime to the succeeding regime, with a tendency to approach a value of 2. The value of k' decreases as n increases. The friction factor—Reynolds' number plot exhibits similarity with that of pipe flow, the friction factor approaching an asymptotic value as the Reynolds' number becomes sufficiently large. A generalised friction factor—Reynolds' number plot for all media is not feasible. The effect of particle orientation on the flow regime is not very significant as per the tests conducted.

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Notations

$A_1, A_2 \ldots\ldots E_2, E_3$ = different gravels tested,

a, b = coefficients,

d = representative dimension of grain-size,

f = Fanning friction factor,

g = acceleration due to gravity,

i = hydraulic gradient,

k = Darcy's coefficient of permeability,

k' = coefficient of non-linear flow,

n = exponent,

R_e = Reynolds' number,

v = mean macroscopic velocity, and

ν = kinematic viscosity.

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