Structural Strength of Granular Materials in Triaxial Compression Test

by

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Notations

$\sigma'_1, \sigma_2', \sigma_3'$ = Effective major, intermediate and minor principal stresses.

σ_m' = Effective mean principal stress.

τ_m = Octahedral shear stress.

$\varepsilon_1, \varepsilon_2, \varepsilon_3$ = Principal strains, axial compression and lateral expansion positive.

v = Volumetric strain, expansion positive.

$\delta\varepsilon_1, \delta\varepsilon_2, \delta\varepsilon_3, \delta v$ = Increments in principal and volumetric strains.

ε_m = Mean principal strain.

γ_m = Octahedral shear strain.

γ = Distortion due to shear stresses.

U = Coefficient of distortional deformation due to shear stress.

V = Coefficient of volumetric deformation due to normal stress.

dE = Total energy involved during shear.

C = A coefficient.

d = Diameter of uniform sphere.

θ = Instantaneous packing angle or the inclination of τ_m with the projection of major principal axis on octahedral plane.

θ_o = Initial packing angle.

δ_1, δ_2 = Deformations of the unit cell in the vertical and horizontal directions.

l_1, l_2 = Instantaneous dimensions of the unit cell in vertical and horizontal directions.

ϕ' = Effective angle of shearing resistance.

ϕ_r' = Corrected angle of inter-particle friction (Bishop).

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ϕ_c' = Corrected angle of inter-particle friction (Ladanyi).

ϕ_f = Corrected angle of inter-particle friction (Rowe *et al*).

ϕ_μ = Angle of inter-particle friction.

ϕ_{cv} = Effective angle of shearing resistance at critical void ratio state.

Introduction

SOIL is a particulate material. In a cohesionless particulate material the shear strength is contributed mainly by two components, namely, the inter-particle friction and particulate structure (*i.e.*, mode of particle packing). In recent years considerable interest has been shown in trying to separate these components of strength. When shear strength of a soil is used in design only single factor of safety is applied to it, rather than different factors of safety to its components. The frictional component within the anticipated stress range may not change significantly but the structural component of strength may be reduced due to time dependent deformation and deterioration of contact material resulting in flow or crushing of the contact material, particularly in tropical soils. The factor of safety applied to the frictional component could be less than that applied to the structural component with an overall reduction in the factor of safety. When such an analysis is adopted, estimation of these components independently is desirable in addition to the study of understanding the mechanism of shear strength generation in granular soils.

The structural strength of a particulate material is influenced by the degree of disorder (randomness), rigidity of groups of particles and porosity of the mass. It also depends on the type of soil, stress path and stress history. The techniques of deposition or compaction of test specimens also influence the structural strength. The structural strength is a characteristic property of particulate materials.

Mohr-Coulomb theory was not developed for a particulate material to account for volume changes and group behaviour of particles ; it predicts the shear resistance between two blocks or particles of material during shear. The model representing this theory is shown in Figure 1 (*a*), in which case the measured effective angle of shearing resistance (ϕ') will be identical to the inter-particle friction angle (ϕ_μ). But when a particulate mass is sheared without volume changes ϕ' is not equal to ϕ_μ. The basic disadvantage of applying this theory to a mass of discrete particles is that it cannot separate the structural strength from the frictional strength. Starting with Taylor, since 1948, attempts have been made by various authors (to mention a few Skemption & Bishop 1950, Penman 1953, Bishop 1954, Roscoe *et al* 1958, Ladanyi 1960, Pooroo-shasb and Roscoe 1961, Rowe *et al* 1964) to relate the strength contributed by friction with Coulomb ϕ of no-volume change condition.

To account for the effect of volume changes Bishop (1954) suggested the following expression which is identical to Mohr-Coulomb expression under no-volume change condition,

$$\sin \phi_r' = \frac{\sigma_1' - \sigma_3' \left(1 + \frac{\delta v}{\delta \epsilon_1}\right)}{\sigma_1' + \sigma'_3} \qquad \ldots(1)$$

but when volume changes occur during shear, component of strength corresponding to volume change is either subtracted or added to the measured $(\sigma'_1 - \sigma'_3)$ depending upon whether the specimen expands or contracts during shear and the value of shear strength corresponding to Mohr-Coulomb condition of no-volume change is obtained ; in this case ϕ_r' is not equal to ϕ_μ. The model representing this case is shown in Figure 1 (*b*). The procedure suggested by Poorooshasb and Roscoe (1961) for separating the components of strength does not take into account the directions of shear stress and strain for obtaining the work involved in shear distortion, although it has the advantage of including the influence of mean principal stress on the structural component of strength. Deviator stress involved in pure shear resistance is

$$(\sigma_1' - \sigma_3') = (\sigma_1' - \sigma_3') - \sigma'_m \left[\frac{\delta v / \delta \varepsilon_1}{1 + \frac{\delta v}{3 \delta \varepsilon_1}} \right] \quad \ldots(2)$$

This equation of Poorooshasb and Roscoe gives values similar to that proposed by Ladanyi (1960), who by considering isotropic and deviatoric stresses separately gave

$$\frac{\sin \phi_c'}{\cos^2 \phi_c'} = \frac{\sin \phi'}{\cos^2 \phi'} - \frac{\frac{\delta v}{\delta \varepsilon_1}}{\left(3 + \frac{\delta v}{\delta \varepsilon_1}\right)} \cdot \frac{3 - \sin \phi'}{2 \cos^2 \phi'} \quad \ldots(3)$$

The method proposed by Rowe *et al* (1964) was derived by minimizing the ratio of input and output energies during shear for a block mechanism of failure when inter-particle friction is developed along the inclined serrated surface of sliding. After accounting for the structural strength they have given

$$\sin \phi_f = \frac{\sigma_1' - \sigma_3' \left(1 + \frac{\delta v}{\delta \varepsilon_1}\right)}{\sigma_1' + \sigma_3' \left(1 + \frac{\delta v}{\delta \varepsilon_1}\right)} \quad \ldots(4)$$

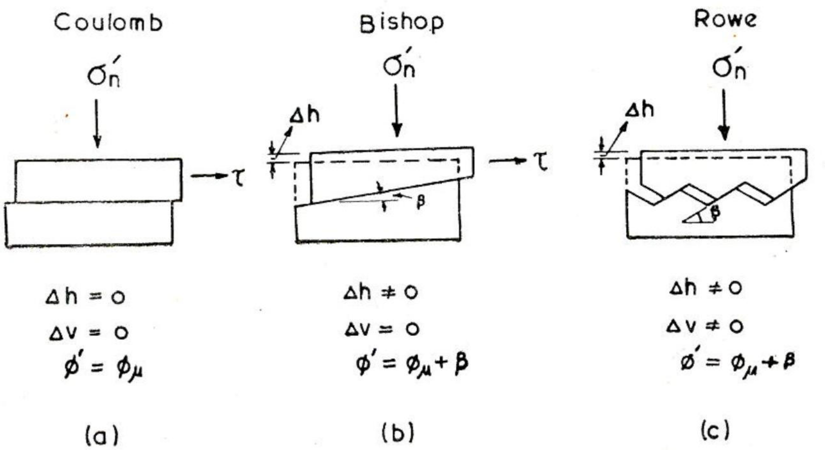

FIGURE 1 (a, b & c) : Simplified models to represent shear strength generation in soils.

The model representing this case is shown in Figure 1 (*c*). Though Rowe (1962) considered an assembly of spherical particles, the mode of failure was essentially of the block type and the failure was implied to take place over the entire mass and not that the failure is initiated at the boundary of the specimen. When block machinism of failure occurs in a dense mass of descrete particles ϕ_f is approximately equal to ϕ_μ. Whereas in a loose mass ϕ_f is greater than ϕ_μ and this is believed to be due to particle reorientation during shear, Rowe *et al* (1964). The above Equations (1) to (4) reduce to Mohr-Coulomb case for no-volume change condition and suggest the absence of structural component of strength. If no structural component of strength exists at no-volume change condition, ϕ measured with effective stresses should correspond to ϕ_μ as in the case of two blocks sliding along a horizontal plane. But in a prticulate material the no-volume change condition during shear is the critical void ratio state in which ϕ_{cv} is approximately equal to $\phi_\mu + 6^0$ (Bishop 1954, Rowe 1964). This suggests that even at no-volume change condition, the structural strength exists in a particulate material and Equations (1) to (4), therefore, enable only partial separation of structural strength.

Of all the four basic packings of spherical particles (Graton *et al* 1935), only hexagonal packing, which is a radially expanding rhombic packing, exists over a wide range of porosity ranging from about 26 to 48 percent, (see Figure 2) and rest of the packings are shortlived. Porosities of most sands fall within these limits. Only hexagonal and quadratic packings (the latter is the radially expanding face-centred cubic packing) undergo three-dimensional mode of failure whereas orthorhombic and

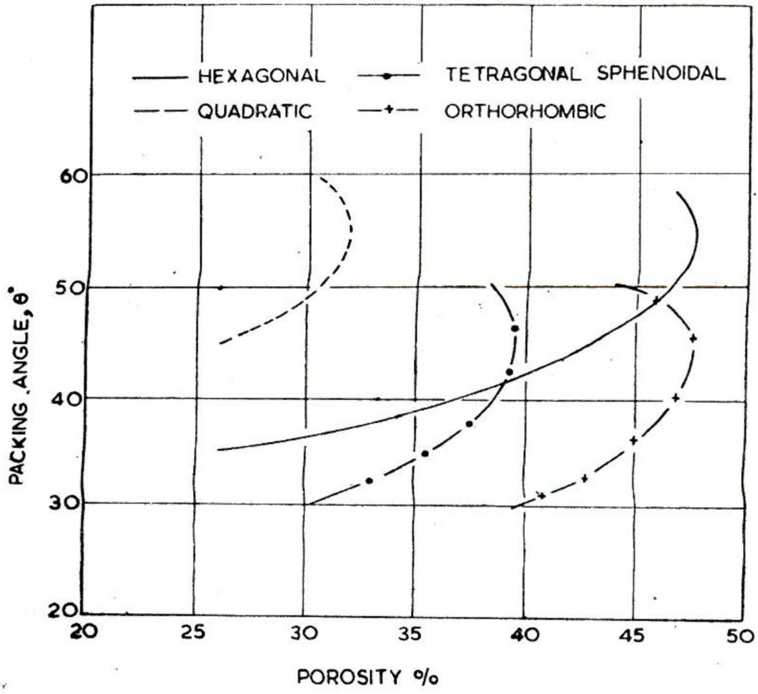

FIGURE 2: Porosity variation in laterally expanding packings of spherical particles.

tetragonal-spheroidal packings exhibit planar failure under triaxial conditions when $\sigma_2=\sigma_3$.

The theoretical relations derived by Thurston *et al* (1959), Rennie (1961), Rowe (1962) and Horne (1965) for face-centred cubic packing give $(\sigma'_1/\sigma'_3)=2$ when $\phi_\mu=0$, whereas the expressions of Dantu (1961), Rowe (1962) and Leussink *et al* (1963) give $(\sigma'_1/\sigma'_3)=4$ for rhombic packing when $\phi_\mu=0$. These effective stress ratios correspond to the structural strength of these packings. The strength component, which remains after substituting $\phi_\mu=0$ in the gross strength expression, is the strength contributed by the particulate structure.

The structural strength of a mass of discrete particles has been derived by assuming the mass consisting the groups of particles forming unit cells representing hexagonal mode of particle packing. It is also assumed that the failure is initiated in the groups of particles on the periphery of the specimen due to loss of two contacts by virtue of their location. The cells located in the interior of the specimen will have 12 contacts in all. The structural strength has also been estimated by considering the energy involved in shear distortion from the strains produced on the octahedral plane assuming the material in the plastic state.

Structural Strength from the Consideration of Particulate Nature of Soil

The entire cohesionless particulate mass is assumed to consist of uniform spherical particles forming groups of hexagonal packing. A unit cell of this packing is considered for derivation of an expression for structural strength.

In Figure 3 let δ_1 and δ_2 be the deformations in the vertical and horizontal directions and let d be the diameter of uniform spheres. Due to these deformations the packing angle θ^0 (the angle between the vertical and the contact line) changes accordingly. From the geometry of packing,

$$\delta_1=2d\,(\cos\theta_0-\cos\theta) \quad ...(5)$$

$$\delta_2=2d\,(\sin\theta-\sin\theta_0) \quad ...(6)$$

where,

θ = instantaneous value of packing angle, and

θ_o = initial value of packing angle-constant.

The increments in δ_1 and δ_2 are given by

$$\triangle\delta_1 = 2\,d\sin\theta\,.\,d\,\theta \quad ...(7)$$

$$\triangle\delta_2 = 2\,d\cos\theta\,.\,d\,\theta \quad ...(8)$$

The instantaneous dimensions in the vertical and horizontal directions are

$$l_1 = 2\,d\cos\theta \quad ...(9)$$

$$l_2 = 2\,d\sin\theta \quad ...(10)$$

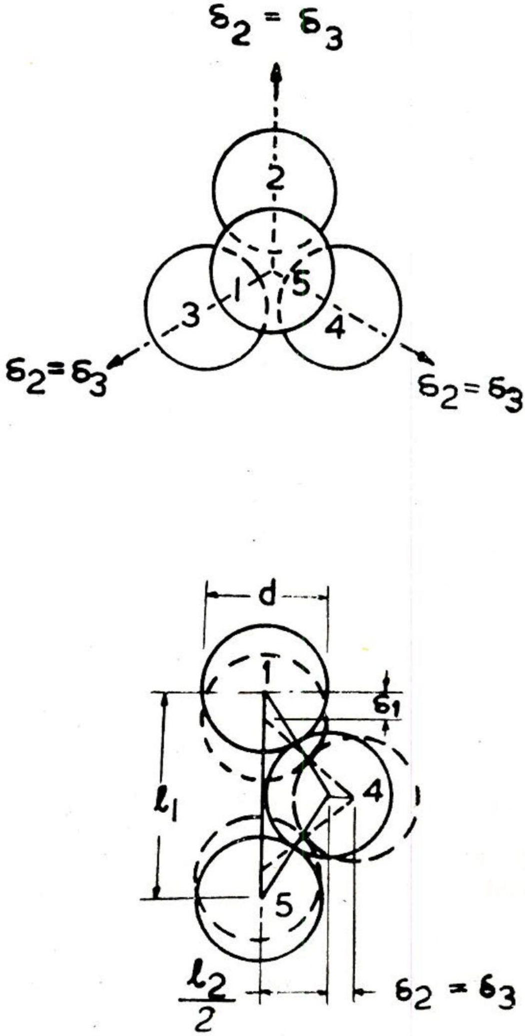

FIGURE 3 : Unit cell representing the mode of hexagonal packing.

Assuming axial compression and lateral expansion as positive, in the case of two-dimensional deformation

$$\frac{\delta\epsilon_1}{\delta\epsilon_2} = \frac{\triangle\delta_1}{l_1} \cdot \frac{l_2}{\triangle\delta_2} = \tan^2\theta \qquad \text{...(11)}$$

In a three-dimensional mode of deformation the strains in the two perpendicular lateral directions for an axisymmetrical case are equal ; therefore

$$\frac{\delta\epsilon_1}{2\,\delta\epsilon_3} = \tan^2\theta = \frac{1}{\left(1+\dfrac{\delta v}{\delta\epsilon_1}\right)} \qquad \text{...(12)}$$

because $\delta\epsilon_1 - 2\,\delta\epsilon_3 = -\delta v$ in which volume expansion is positive. The theoretical strength equation (derived by the author for a laterally expanding hexagonal packing, see Appendix) is given by

$$\frac{\sigma'_1}{\sigma'_3} = \frac{1}{\tan\theta \, . \, \tan(\theta - \phi_\mu)} \qquad \ldots(36)$$

Substituting $\phi_\mu = 0$ in Equation (36) the strength contributed by the particulate structure is

$$\left(\frac{\sigma_1'}{\sigma_3'}\right)_s = \frac{1}{\tan^2\theta} \qquad \ldots(13)$$

Therefore,

$$\left(\frac{\sigma_1'}{\sigma_3'}\right)_s = \left(1 + \frac{\delta v}{\delta \epsilon_1}\right) = \frac{1}{\tan^2\theta} \qquad \ldots(14)$$

Even by substituting $\phi_\mu = 0$ in Rowe's minimum energy ratio (1962)

$$\frac{\sigma_1'}{\sigma_3'\left(1 + \frac{\delta v}{\delta \epsilon_1}\right)} = \tan^2\left(45 + \frac{\phi_\mu}{2}\right) \qquad \ldots(15)$$

the structural strength

$$\left(\frac{\sigma_1'}{\sigma_3'}\right)_s = \left(1 + \frac{\delta v}{\delta \epsilon_1}\right)$$

which is same as given by Equation (14). By putting $\theta = 35.26°$ for the densest state of rhombic packing in Equation (14), $\left(1 + \frac{\delta v}{\delta \epsilon_1}\right) = 2.$

Horne (1965) in his Figure 8, for a disordered material of spherical particles showed effective stress ratio at maximum dilatancy rate approximately equal to 2 for $\phi_\mu = 0$. This is in confirmation with the author's Equation (14).

Equation (14) suggests that by subtracting the dilatancy rate from the observed value of effective stress ratio, the stress ratio $(\sigma_1'/\sigma_3')_c$ corresponding to the strength contributed by the frictional component is given as

$$\left(\frac{\sigma_1'}{\sigma_3'}\right)_c = \left(\frac{\sigma_1'}{\sigma_3'}\right) - \left(1 + \frac{\delta v}{\delta \epsilon_1}\right).$$

In terms of Mohr-Coulomb ϕ after separating the structural component of strength

$$\sin\phi_\mu = \frac{(\sigma_1'/\sigma_3')_c - 1}{(\sigma_1'/\sigma_3')_c + 1} \qquad \ldots(16)$$

$$i.e., \quad \sin\phi_\mu = \frac{\left[\frac{\sigma_1'}{\sigma_3'} - \left(1 + \frac{\delta v}{\delta \epsilon_1}\right)\right] - 1}{\left[\frac{\sigma_1'}{\sigma_3'} - \left(1 + \frac{\delta v}{\delta \epsilon_1}\right)\right] + 1} \qquad \ldots(17)$$

Equation (17) suggests that when Mohr-Coulomb relation

$$\sin \phi_{\mu} = \frac{(\sigma_1'/\sigma_3') - 1}{(\sigma_1'/\sigma_3') + 1}$$

for two blocks sliding on a plane inclined to the major principal plane at an angle of $45+\phi_{\mu}/2$, refers to the case of K_o-consolidation when $\sigma_1' > \sigma_3'$ and $\delta\epsilon_1 = -\delta v$ for a particulate material. By substituting $\delta v = 0$ in Equation (17), it does not reduce to Mohr-Coulomb expression suggesting the presence of structural strength even at no-volume change condition.

Figure 4 shows the individual contribution to strength by the particulate structure and the frictional resistance and their variation with porosity for a mass of hexagonal packing in axisymmetrical triaxial compression test. One striking feature in this figure is its resemblence with the shapes of the experimental curves obtained by Bjerrum *et al* (1951). Whereas most authors (Bishop *et al* 1950, Penman 1953, and others) have obtained experimentally concave upward curves which when extended suggest negligible variation of strength even for porosities greater than the maximum. In fact the curves should indicate rapid decrease of strength in the range of maximum porosity due to instability of the particulate structure. This is suggested by the theoretical analysis. In the range of maximum porosity, dense columner type of structure is developed along the periphery of the sample due to contraction of rubber membrane or disturbance in the preparation of the specimen

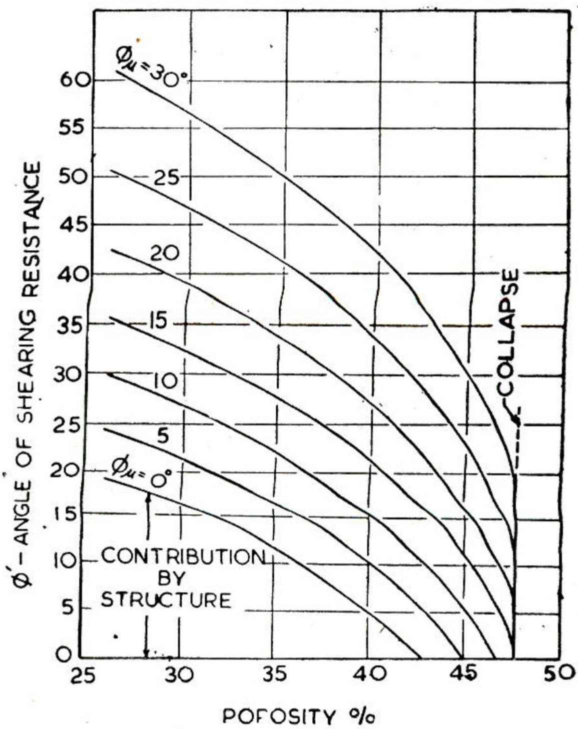

FIGURE 4 : Strength components in a laterally expanded hexagonal packing.

and variation of porosity exists from surface to interior of the specimen. As a result of this concave upward curves are obtained. When loosest specimens are prepared in very thin rubber membrane and tested under very low cell pressure concave downward curves are obtained particularly if the cohesionless soil is deposited in upward flowing water as adopted by Bjerrum *et al* (1961).

Structural Strength from Stresses and Strains Produced on the Octahedral Plane

The general theory, proposed by Hoshino (1948, 1951 and 1957) for soils on the assumption that at any point of a stressed mass the amount of energy governs the deformation in the range from elastic state to plastic failure as well as the final stress condition at failure, is adopted in estimating the structural strength in axisymmetrical triaxial compression test.

Considering an octahedral plane (Figure 5) at any point of a stressed mass, the forces acting on the plane are mean normal stress σ_m' and shear stress τ_m. The latter is inclined to the projection of major principal axis at an angle θ. The three principal stresses may be given as

$$\left.\begin{aligned} \sigma_1' &= \sigma_m' + \sqrt{2}\,\tau_m \cos\theta \ldots\ldots\ldots \\ \sigma_2' &= \sigma_m' + \sqrt{2}\,\tau_m \cos\left(\tfrac{2}{3}\pi - \theta\right) \ldots \\ \sigma_3' &= \sigma_m' + \sqrt{2}\,\tau_m \cos\left(\tfrac{2}{3}\pi + \theta\right) \ldots \end{aligned}\right\} \quad \ldots(18)$$

where

$$\sigma_m' = \tfrac{1}{3}\left(\sigma_1' + \sigma_2' + \sigma_3'\right)$$

$$\tau_m = \tfrac{1}{3}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

For axisymmetrical case where $\sigma_2 = \sigma_3$, the directional angle θ of τ_m becomes zero.

Therefore,

$$\left.\begin{aligned} \sigma_1' &= \sigma_m' + \sqrt{2}\,\tau_m \quad \ldots \\ \sigma_2' &= \sigma_3' = \sigma_m' - \frac{1}{\sqrt{2}}\,\tau_m \ldots \end{aligned}\right\} \quad \ldots(19)$$

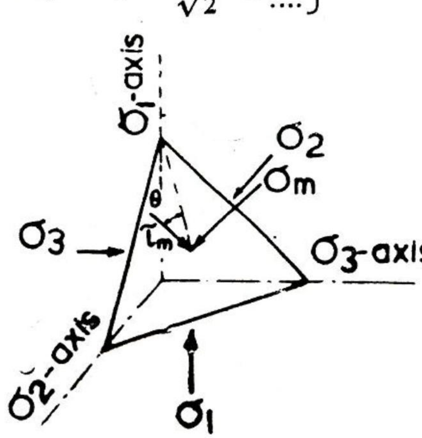

FIGURE 5 : Octahedral plane.

where

$$\sigma_m' = \tfrac{1}{3}(\sigma_1' + 2\,\sigma_3')$$

$$\tau_m = \sqrt{2}/3\,(\sigma_1 - \sigma_3)$$

The increments in principal strains in terms of σ_m' and τ_m are given as follows assuming compressive strains positive,

$$\left.\begin{aligned} \frac{d\epsilon_1}{1-\epsilon_1} &= \frac{1}{3V}\,d\,\sigma_m + \frac{1}{3U}\,d\,\tau_m \quad \ldots \\ \frac{d\epsilon_2}{1-\epsilon_2} &= \frac{d\epsilon_3}{1-\epsilon_3} = \frac{1}{3V}\,d\,\sigma_m - \frac{1}{3\sqrt{2}\,U}\,d\,\tau_m \ldots \end{aligned}\right\} \quad \ldots(20)$$

where

V = coefficient of volumetric deformation due to normal stress.

U = Coefficient of distortional deformation due to shear strain.

The increment in the total energy per unit volume due to small change of strains is given by

$$dE = \frac{d\epsilon_1}{1-\epsilon_1}\sigma_1' + \frac{2\,d\epsilon_3}{1-\epsilon_3}\sigma_3' \quad \ldots(21)$$

Substituting the values from Equations (19) and (20) in Equation (21)

$$dE = \frac{\sigma_m'}{V}\,d\sigma_m + \frac{\tau_m}{U}\,d\,\tau_m \quad \ldots(22)$$

With the assumption that the coefficients of deformation due to compression and distortion are functions of corresponding energies supplied to the mass, Hoshino (1957) showed that the increment of energy due to pure shear when σ_m' is constant is given from Equation (22) as

$$dE_s = \frac{\tau_m}{U}\,d\,\tau_m \quad \ldots(23)$$

The volume changes and shear distortion in a unit mass are given by

$$\left.\begin{aligned} 1-v &= (1-\epsilon_m)^3 \ldots \\ 1-\gamma &= (1-\gamma_m)^3 \ldots \end{aligned}\right\} \quad \ldots(24)$$

Similar to Equation (20) when σ_3' is constant

$$\frac{d\gamma}{1-\gamma} = \frac{d\,\tau_m}{U} \quad \ldots(25)$$

where

v = volume change per unit volume due to normal stress,

γ = distortion per unit, volume due to shear stress,

ϵ_m = mean of compressive strains due to normal stresses, and

γ_m = mean shear strain due to shear stresses.

For axisymmetrical case

$$\left.\begin{aligned} \epsilon_m &= \tfrac{1}{3}(\epsilon_1 + 2\,\epsilon_3) \quad \ldots \\ \gamma_m &= \frac{\sqrt{2}}{3}(\epsilon_1 - \epsilon_3) \ldots \end{aligned}\right\} \qquad \ldots(26)$$

By re-arranging Equation (26)

$$\gamma_m = \frac{\epsilon_1 - \epsilon_m}{\sqrt{2}} \qquad \ldots(27)$$

Substituting the values of γ_m and ϵ_m from Equation (24) in Equation (27) and re-arranging

$$1-\gamma = \left[1 - \frac{\epsilon_1 - \{1-(1-v)^{1/3}\}}{\sqrt{2}}\right]^3 \qquad \ldots(28)$$

By expanding Equation (28) and eliminating the 2nd order terms

$$(1-\gamma) = 1 - \frac{3}{\sqrt{2}}(\epsilon_1 - \tfrac{1}{3}v) \qquad \ldots(29)$$

From Equations (23) and (25), the shear stress producing shear distortion alone is given by

$$\frac{dE_s}{d\gamma} = \frac{\tau_m}{1-\gamma} \qquad \ldots(30)$$

Substituting the value of $(1-\gamma)$ from Equation (29) in Equation (30) and re-arranging and assuming expansion of soil mass positive

$$\frac{dE_s}{d\gamma} = \frac{2\ (\sigma_1 - \sigma_3)}{3(\sqrt{2} - 3\epsilon_1 - v)} \qquad \ldots(31)$$

$$= C\,(\sigma_1 - \sigma_3)$$

where

$$C = \frac{0.667}{(\sqrt{2} - 3\epsilon_1 - v)} \qquad \ldots(32)$$

The shear stress involved in shear distortion is due to the frictional resistance which is given by Equation (31). The deviator stress associated with structural strength is

$$(\sigma_1-\sigma_3)\left[1-\frac{2}{3\,(\sqrt{2}-3\,\epsilon_1-v)}\right] \quad ...(33)$$

The frictional resistance may be related to Mohr-Coulomb ϕ after separating the structural strength from the measured deviator stress as

$$\sin\phi_\mu=\frac{C(\sigma_1-\sigma_3)}{C(\sigma_1-\sigma_3)+2\,\sigma'_3} \quad ...(34)$$

Experimental Evidence

Consolidated-drained triaxial compression tests were conducted on saturated specimens of Badarpur sand according to the procedure described by Bishop and Henkel (1957). The experiments were run in a constant temperature room and volume changes were measured using series of micropipettes. Evaporation of water in the micropipettes was minimized using xylene (Barden *et al* 1966). These tests were conducted under low confining pressure of 0.7 kg/cm² to avoid crushing of particles.

In Figures 6, 7 and 8 the development of ϕ_μ with axial strain is compared for the three approaches represented by Equations (4), (17) and (34). From the method suggested by Rowe *et al* the values of ϕ after separating the structural strength are higher for looser specimens than for the denser ones, whereas, from the two approaches presented by the author

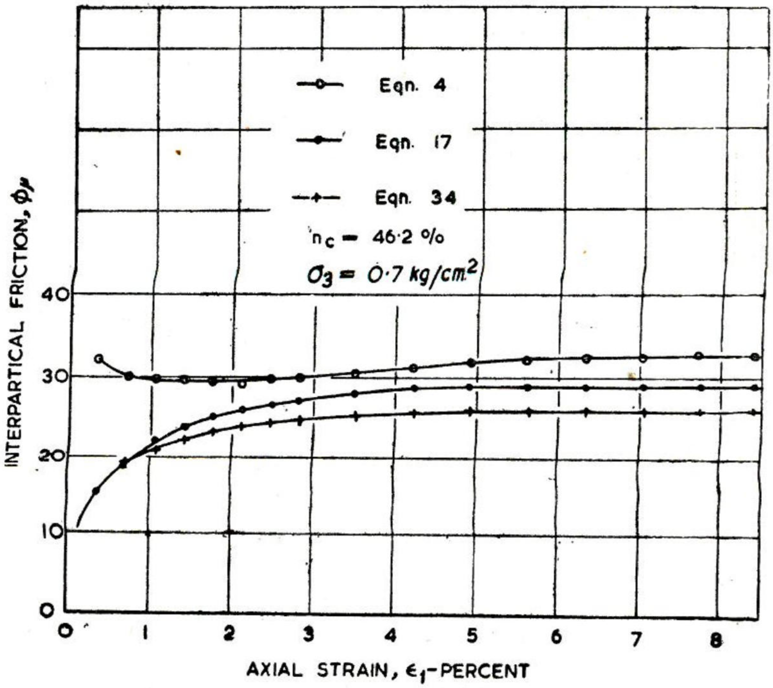

FIGURE 6 : Development of inter-particle friction with strain.

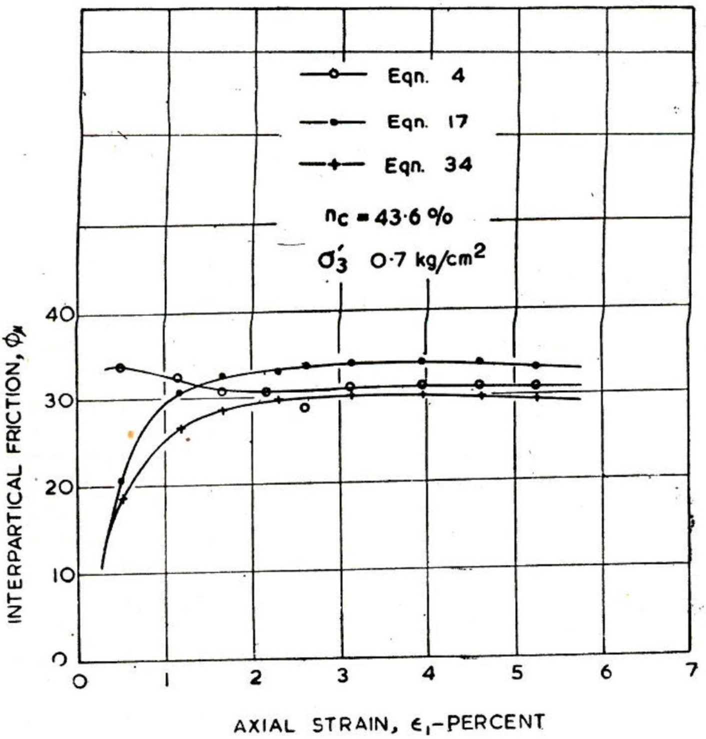

FIGURE 7 : Development of inter-particle friction with strain.

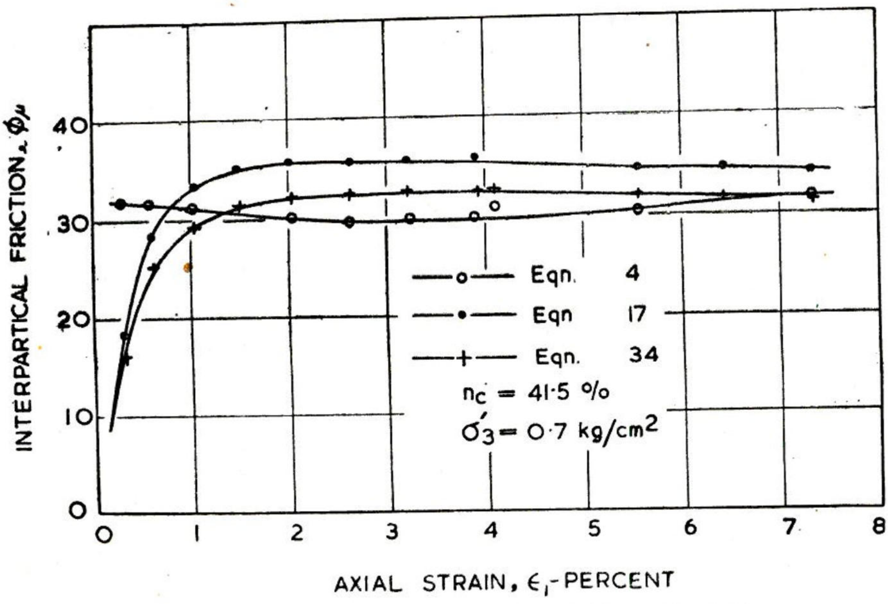

FIGURE 8 : Development of inter-particle friction with strain.

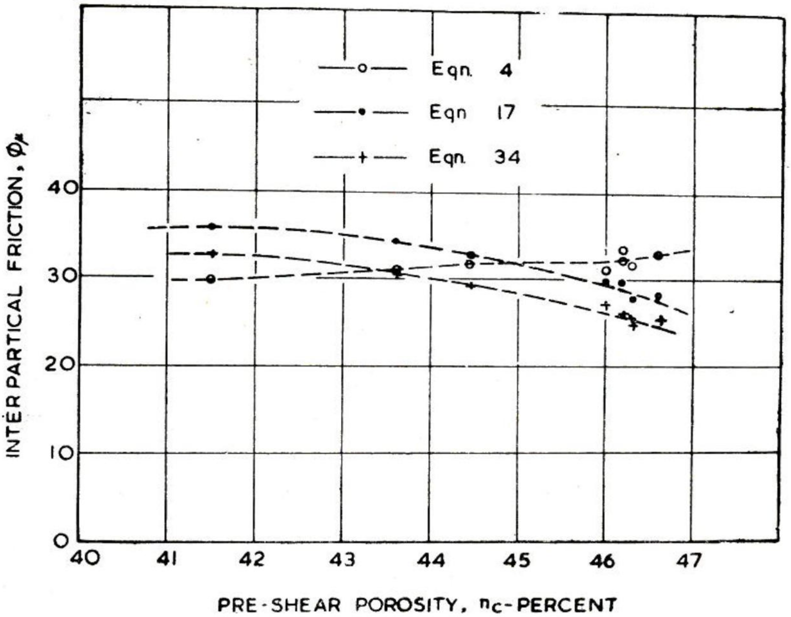

FIGURE 9 : Comparison of theories for the prediction of inter-particle friction for Badarpur sand.

ϕ, after separating the structural strength decreases with increasing porosity (Figure 9). This may be due to the rotation of particles in range of high porosities when lower ϕ_μ is mobilised. The nature of variation of ϕ_μ with strain in these two approaches is very similar. The particulate theory of rigid spherical particles gives higher value of ϕ_μ than the plastic theory by about 3° at and beyond failure. This difference may be due to the assumptions involved in these theories.

Conclusions

The components of strength due to inter-particle friction and particulate structure have been separated by two different approaches, namely particulate and plastic nature of material. In the former case, a laterally expanding hexagonal packing of spherical particles is assumed to represent the behaviour of sand and the failure is assumed to start from the periphery of the specimen tested in triaxial compression, *i.e.*, the stability of a unit cell which looses two contacts by virtue of its location at the periphery, is considered. Other investigators considered the stability of a cell located at the centre of the specimen having in all 12 particle contacts and the mode of failure assumed was block-type and not a laterally expanding lattice. By adopting Hoshino's generalised theory for plastic failure of soils, structural strength has been estimated by separating the energy involved in shear distortion. Both the theories suggest that the structural strength as well as frictional strength decrese with increase of porosity, unlike the variation suggested by Rowe *et al.* The particulate theory of rigid spherical particles estimates higher value of inter-particle friction than the plastic theory by about 3° at and beyond failure. But these theories suggest that structural strength exists even at no-volume change condition during shear distortion. The nature of variation of inter-particle friction in both the theories is essentially similar. The particulate theory predicts concave downward curve between strength and porosity represented on ordinate and abscissa respectively and this is supported by experimental evidence.

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APPENDIX

Strength of a particulate material should take into consideration the friction at the sliding contacts, group behaviour of particles, mode of particle packing, number of contacts and shape characteristics of particles. In the present approach the effect of the shape of particles on strength is not included in the theoretical analysis. Hexagonal packing of spherical particles is the only stable and predominantly occurring packing which can exist over a wide range of porosity. A derivation for the strength of this packing is given in this section.

Assumptions

(1) The particles are assumed to be rigid and spherical in shape.

(2) The entire particulate mass consists of packings of the groups of particles forming unit cells representing the basic characteristics of the packing.

(3) Particles in the interior of the mass have 12 particle contacts each at the closest packing, whereas the particles at the surface of the specimen enclosed in a rubber membrane loose some contacts. Through these lost contacts σ_3 is applied to the particle through the rubber membrane. Though the mass may be uniformly packed forming an ordered packing but at the boundary the groups are weaker and the failure is intiated at the boundary of the specimen. Therefore failure of a group at the surface is assumed to be the failure of the entire specimen.

(4) In the triaxial case when $\sigma_2=\sigma_3$ failure of groups takes place by uniform movement of particles radially in the lateral directions.

Derivation for the Strength of Hexagonal Packing

In Figure 10 (*a*), a hexagonal packing (which is a radially expanding rhombic packing) is shown in plan and elevation with a unit cell representing a mode of packing between the maximum and minimum porosities for this group. The particles forming this group are numbered from 1 to 5. When this group is at the closest packing, the particles 2,3 and 4 are in contact with each other but when sliding occurs due to axial loading these three particles are pushed radially outward and loose contact

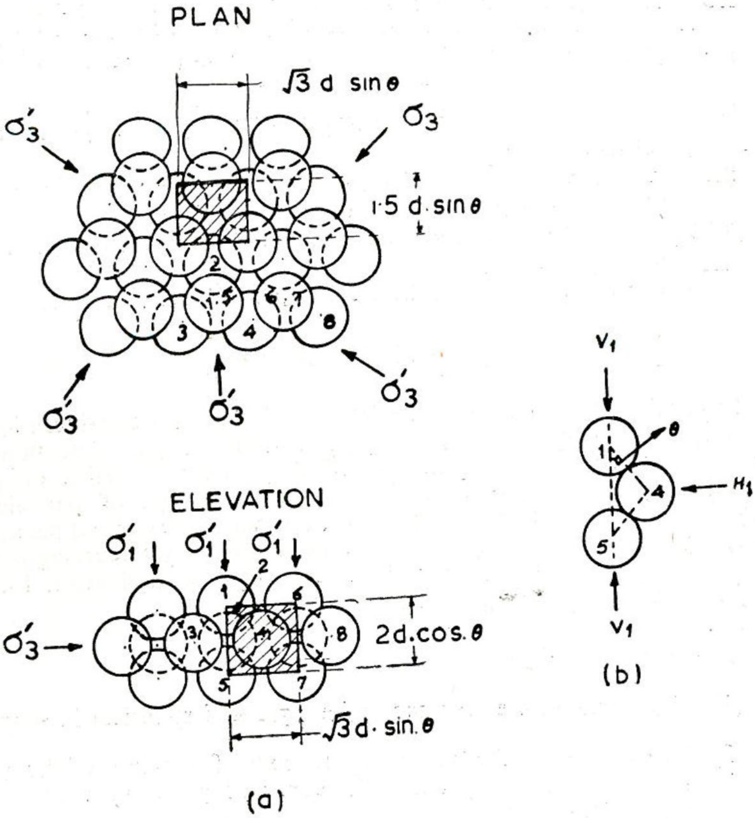

FIGURE 10 (a and b) :

(a) Plan and elevation of a hexagonal packing.

(b) Arching of particles.

with each other and form arches as shown in Figure 10(*b*). This condition exists under uniform deformation until the packing attains the maximum porosity. During this process each particle makes totally six contacts with the neighbouring particles. Considering one of the particles, say particle 3, it acquires a horizontal support from its neighbouring particles through two contacts (in a verticle plane) for its stability while sliding is experienced at the other four contacts. *This condition also exists at the boundary of the sample.* Therefore, for the stability of the group, the horizontal force should be applied equal in magnitude and opposite in direction to the resultant of the horizontal forces at the four sliding contacts. The vertical force on each particle in the horizontal layer is distributed over three contacts.

Now the condition for the particle 3 to slide at a contact is

$$\frac{V_1}{H_1}=\frac{1}{\tan(\theta-\phi_\mu)} \qquad \ldots(35)$$

where V_1=vertical force at the contact,

H_1=horizontal force at the contact,

θ=packing angle in degrees (it is the acute angle between the vertical and the contact line), and

ϕ_μ=friction angle at the contact.

Let V be the vertical force acting on each particle in the horizontal plane and H be the horizontal force required by each particle in the vertical plane for equilibrium. If A_1 and A_2 are the areas on which V and H act respectively, then from the geometry of the packing

$$A_1 \text{ (on which } V \text{ acts)} = (\sqrt{3}\, d \,.\, \sin\theta)\,(1{\cdot}5\, d \,.\, \sin\theta)$$

$$= 1{\cdot}5\sqrt{3}\, d^2 \,.\, \sin^2\theta$$

$$A_2 \text{ (on which } H \text{ acts)} = (2\, d \,.\, \cos\theta)\,(\sqrt{3}\, d \,.\, \sin\theta)$$

$$= 2\sqrt{3}\, d^2 \,.\, \sin\theta \,.\, \cos\theta$$

$$V_1 = V/3 \text{ and } H_1 = H/4$$

$$V = \sigma_1' \,.\, A_1 \text{ and } H = \sigma_3' \,.\, A_2$$

where d = diameter of particle

σ_1' and σ_3' = effective axial and radial stresses respectively.

$$\therefore \quad \frac{V_1}{H_1} = \frac{4V}{3H} = \frac{4 \,.\, \sigma_1' \,.\, A_1}{3 \,.\, \sigma_3' \,.\, A_2} = \frac{1}{\tan(\theta - \phi_\mu)}$$

$$\therefore \quad \frac{\sigma_1'}{\sigma_3'} = \frac{1}{\tan\theta \,.\, \tan(\theta - \phi_\mu)} \qquad \text{...(36)}$$

The last expression gives the strength of a hexagonal packing. At the closest packing when $\theta = 35{\cdot}26°$, this expression gives the strength of a rhombic packing *provided that the particles have the freedom to slide in the radial directions* when axial stress is applied. The author's work (Ramamurthy 1966) has indicated that the particles except in the closest state generally have freedom to slide in radial directions and the mode of failure is essentially by expansion of lattice radially in the lateral directions.