Comparison between the Bishop Method and the Method of Characteristics for Stability Analysis of Slopes

by

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Introduction

THIS investigation is concerned with the comparison of the Bishop's method and the method of characteristics, for the stability analysis of homogeneous earth slopes. For a slope whose horizontal top surface is subjected to a uniform normal surcharge (Figure 1), the contour of the slope can be determined using the method of characteristics by knowing the soil parameters (Sokolovsky, 1965). Since the slope thus determined is in a limiting state of equilibrium, its factor of safety is unity. The factor of safety of this slope can also be determined by Bishop's method (Bishop, 1955). Thus a comparison can be made between the two approaches.

In finding the factor of safety by Bishop's method, the surcharge q is taken into consideration by adding to the weight of each of the slices to the right of z-axis, the corresponding surcharge. Further, the contour of the slope obtained by the method of characteristics extends downwards indefinitely as in Figure 1. However, for purposes of this study the height of the slope can be fixed at a certain depth below the top surface, corresponding to the lowest point on the contour of the slope as obtained by the method of characteristics (Figure 2).

Analysis

DETERMINATION OF THE CONTOUR OF THE SLOPE BY THE METHOD OF CHARACTERISTICS

The analysis used herein is the same as that presented by Sokolovsky (1965). The soil adjacent to the slope shown in Figure 1 is in plastic equilibrium. It has strength parameters c and ϕ and the tangent to the slope at point O makes an angle β_o with x-axis. Along the positive x-axis a uniform normal surcharge, q, acts. The equivalent surcharge p_0, along Ox is then

$$p_0 = q + H \qquad \ldots(1)$$

in which, $H = c \cot \phi$.

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In arriving at non-dimensional quantities, the stresses are divided by a characteristic stress equal to cohesion, c, and the distances are divided by a characteristic length equal to c/γ, in which γ = unit weight of the soil. Hence, the non-dimensional mean stress at any point, $\sigma^* = \sigma/c$, in which

$$\sigma = \frac{\sigma_x + \sigma_z}{2} + H,$$

and, σ_a, σ_z = the normal stresses in x and z directions, respectively.

Along the boundary OC of the Rankine zone the values of σ^* and ψ are given by (Sokolovsky, 1965, Harr, 1966)

$$\sigma^* = \frac{z'+P}{1+\sin\phi}, \text{ and } \psi = \frac{\pi}{2} \qquad ...(2)$$

in which, ψ = inclination of major principal stress to x-axis, z' = non-dimensional z-coordinate and P = non-dimensional equivalent surcharge = p_0/c.

Along the contour of the slope, the known conditions are

$$\sigma^* = \frac{H'}{1-\sin\phi}, \quad \frac{dz'}{dx'} = \tan\beta, \text{ and } \psi = \beta \qquad ...(3)$$

in which, H' = cot ϕ, and β = inclination of the slope with horizontal, at any point.

At the singular point O, ψ changes from $\pi/2$ just to the right of O to the value β_0 just to its left.

The angle β_0 is determined from the following relationship (Harr, 1966) which can be derived from the boundary conditions, *viz.*, Equations (2) and (3) :

$$\beta_o = \frac{\pi}{2} + \frac{\phi}{2} \log_e \left(\frac{p_o}{H} \frac{1-\sin\phi}{1+\sin\phi} \right) \qquad ...(4)$$

In order that zones I and II do not overlap (Figure 1), β_0 should be greater than $\pi/2$, and this condition leads to the inequality condition :

$$q \geqslant H\left(\frac{1+\sin\phi}{1-\sin\phi} - 1 \right)$$

The dimensionless minimum surcharge which corresponds to $\beta_0 = \pi/2$, is therefore given by

$$q' = \frac{q}{c} = \frac{2\cos\phi}{1-\sin\phi} \qquad ...(5)$$

If any value of q higher than this minimum value is specified, the corresponding value of β_0 can be easily obtained from Equation (4).

With all the boundary conditions thus determined, the next step is to apply the relationships along the two families of slip lines in the zone of rupture. Writing the equations along the characteristics in finite difference form (Sokolovsky, 1965), the following expressions are obtained

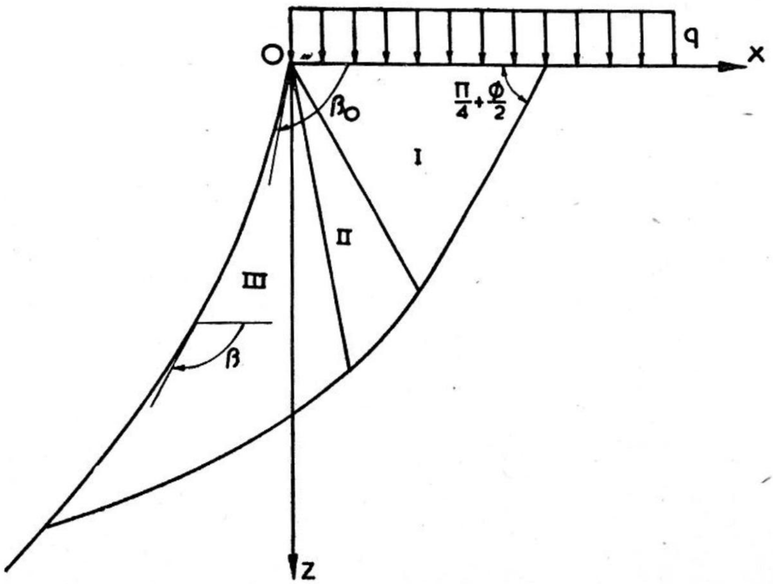

FIGURE 1 : Zones of rupture of a slope in critical equilibrium.

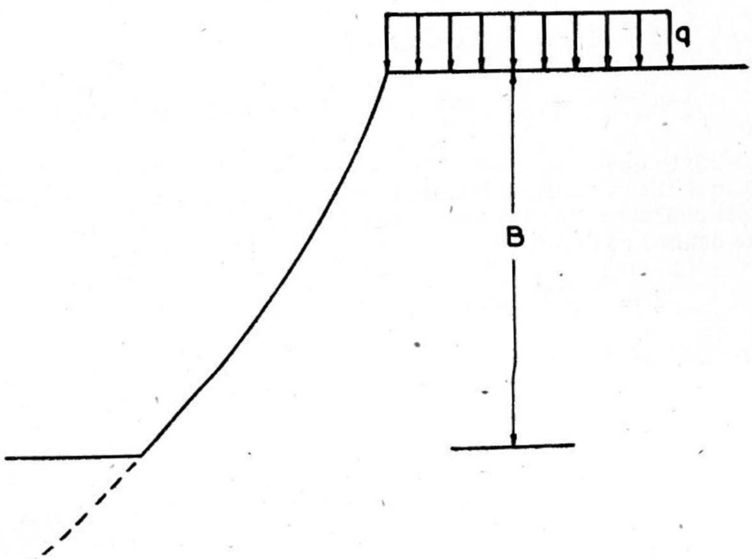

FIGURE 2 : Slope with finite height.

for determining the quantities at a point C from known points A and B (Figure 3) :

$$x' = \frac{x_2' \tan(\psi_2-\mu) - z_2' - x_1' \tan(\psi_1+\mu) + z_1'}{\tan(\psi_2-\mu) - \tan(\psi_1+\mu)} \quad \ldots(6)$$

$$z' = z_1' + (x'-x_1') \tan(\psi_1+\nu) \quad \ldots(7)$$

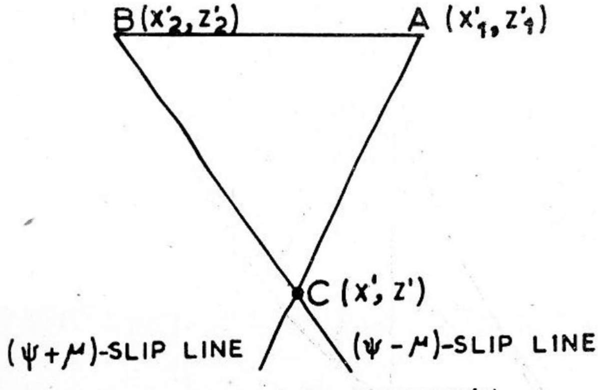

FIGURE 3 : Calculations for a new point.

$$\xi = \xi_1 + e_i\,(x' - x_1') \qquad \text{...(8)}$$

$$\eta = \eta_2 + e_j\,(x' - x_2') \qquad \text{...(9)}$$

in which,

$$e_i = \frac{\cos(\psi_1 - \mu)}{2\,\sigma_1^*\,\sin\phi\,\cos(\psi_1 + \mu)} \qquad \text{...(10)}$$

$$e_j = \frac{\cos(\psi_2 + \mu)}{2\,\sigma_2^*\,\sin\phi\,\cos(\psi_2 - \mu)} \qquad \text{...(11)}$$

$\mu = \frac{\pi}{4} - \frac{\phi}{2}$ and x' = non-dimensional x-coordinate.

In the above equations, quantities with subscript 1 are those at the point A and quantities with subscript 2 are those at B. ξ and η are the variables characterising the variations of σ^* and ψ along the slip lines and are defined as follows :

$$\xi = \frac{\cot\phi}{2}\log_e \sigma^* + \psi$$

$$\eta = \frac{\cot\phi}{2}\log_e \sigma^* - \psi$$

For determining the quantities x', z', ψ and σ^* for points along the contour of the slope, Equation (3) is used along with the two equations along a slip line of the $(\psi+\mu)$—family which intersects the slope. The expressions obtained in this case after writing the equations in finite difference form and solving, are

$$x' = \frac{x_1'\tan(\psi_1+\mu) - z_1' - x_2'\tan(\psi_2) + z_2'}{-\tan\psi_2 + \tan(\psi_1+\mu)} \qquad \text{...(12)}$$

$$z' = z_2' + (x' - x_2')\tan(\psi_2) \qquad \text{...(13)}$$

and

$$\psi = \xi_1 + e_i\,(x' - x_1') - \frac{\cot\phi}{2}\log_e \sigma^* \qquad \text{...(14)}$$

σ^* is obtained directly from Equation (3) by dividing by the characteristic stress, c.

The numerical solution of the problem is carried out by starting from the boundary OC of the Rankine zone (Figure 1). The values of x' and z' thus determined define the contour of the slope.

DETERMINATION OF FACTOR OF SAFETY BY BISHOP'S METHOD

The slope shown in Figure 2, whose contour can be determined as explained above, is analysed for finding its factor of safety, by the Bishop's method. The expression given by Bishop (1955) for determining the factor of safety for any assumed circular rupture surface (Figure 4) is

$$F = \frac{\Sigma\,[cb+\tan\phi\,(W-ub)]\,1/M_\alpha}{\Sigma\, W \sin\alpha} \qquad \text{...(15)}$$

in which,

F = factor of safety for the assumed slip circle,

W = weight of any slice = $bh\,\gamma$,

h = height of the slice at its centre,

α = angle defined in Figure 4,

c, ϕ = effective strength parameters,

b = width of slice,

u = pore pressure at the bottom of the slice, and

$$M_\alpha = \frac{1+(\tan\alpha\,\tan\phi)/F}{\sec\alpha} \qquad \text{...(16)}$$

Assuming a characteristic length, $l = c/\gamma$, and introducing the dimensionless variables

$$b' = \frac{b}{l},\; h' = \frac{h}{l},\; \bar{B} = \frac{u}{\gamma h} \qquad \text{...(17)}$$

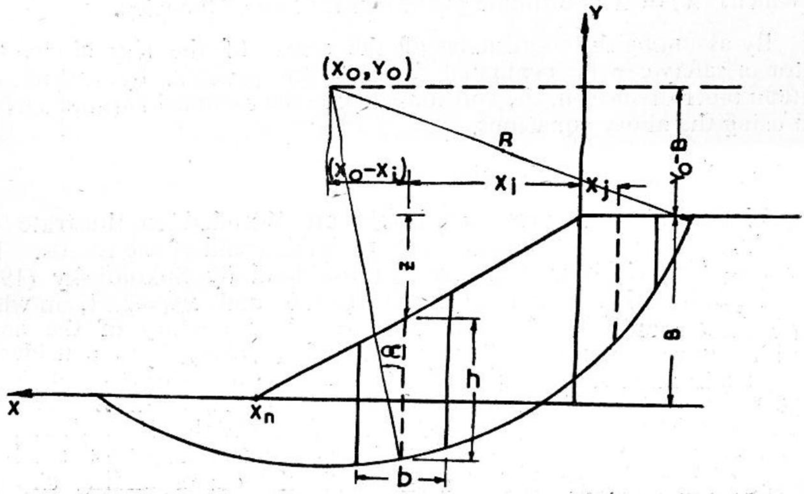

FIGURE 4: Analysis by method of slices.

Equation (15) is written as

$$F = \frac{\Sigma\, b'\,[1+(1-\overline{B})\, h' \tan\phi]\, 1/M_\alpha}{\Sigma\, b'\, h' \sin\alpha} \qquad (18)$$

The pore pressure coefficient, $\overline{B}$, in the above equation is assumed to be fairly constant over the cross-section of the slope. The mean height of the slice, h, is expressed as (Figure 4)

$$h = \sqrt{R^2-(X_0-X_i)^2} - (Y_0-B)-z \qquad \text{...(19)}$$

in which,

R = radius of slip circle,

X_0, Y_0 = coordinates of the centre of slip circle,

B = height of the slope,

X_i = X-coordinate of the centre of the slice, and

z = the depth of the slice below the horizontal top surface of the slope.

Since only toe circles are assumed herein as failure surfaces, R can be obtained easily knowing X_0 and Y_0, since

$$R^2 = Y_0^2 + (X_n-X_0)^2 \qquad \text{...(20)}$$

in which, X_n = horizontal distance of toe of slope from origin of X, Y coordinate system.

Therefore, the factor of safety given by Equation (18) is found to be a function of only two variables X_0 and Y_0, the other parameters, ϕ and $\overline{B}$ being constants. The expression for the mean height of slice in the portion to the right of the Y-axis (Figure 4) is

$$h = \sqrt{R^2-(X_0-X_j)^2} - (Y_0-B) \qquad \text{...(21)}$$

in which, X_j = X-coordinate of the central line of the slice.

By assuming the coordinates of the centre of the slip circle, the factor of safety can be evaluated for a given problem by assuming a finite number of slices in the soil mass above the assumed rupture surface and using the above equations.

Results and Discussion of Results

Numerical results presented herein are intended to illustrate the procedure used and to indicate the probable trend of the results. The following data which are the same as those used by Sokolovsky (1965) are adopted in this numerical work : $\phi=30°$ and $x_M=2.31$, in which x_M is the x-coordinate of the last point on the boundary of the active Rankine zone (Figure 5). Two values of surcharge are considered : (*i*) The minimum value of q' as given by Equation (5), which is equal to 3·4641, (*ii*) $q' = 4{\cdot}4641$. For these two cases, from Equation (4), $\beta_o = 90°$ and $102°36'$, respectively. The contours of the slopes determined by the method of characteristics for the above two cases of surcharge are shown in Figures 5 and 6 along with the corresponding slip line fields.

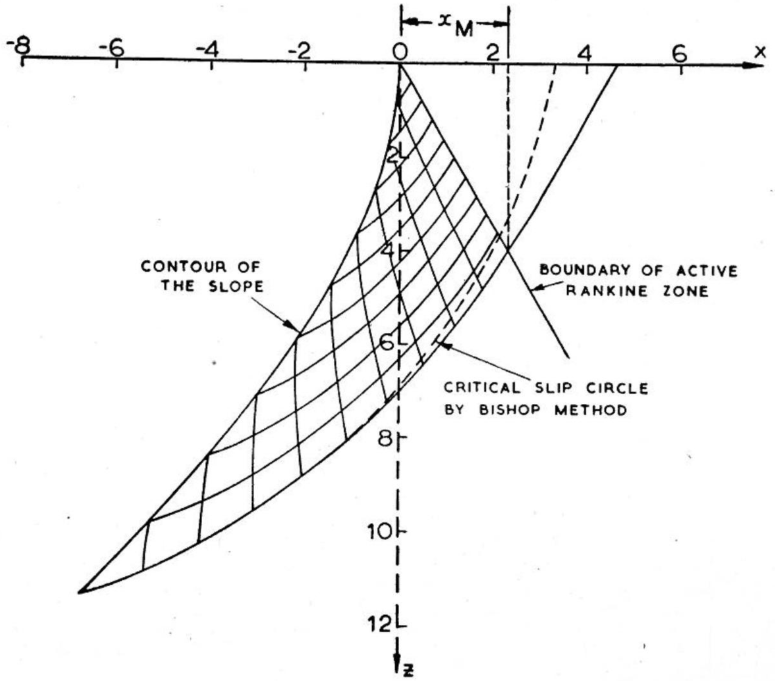

FIGURE 5 : Contour of slope and slip lines for $\phi = 30°$ and $q' = 3{\cdot}4641$.

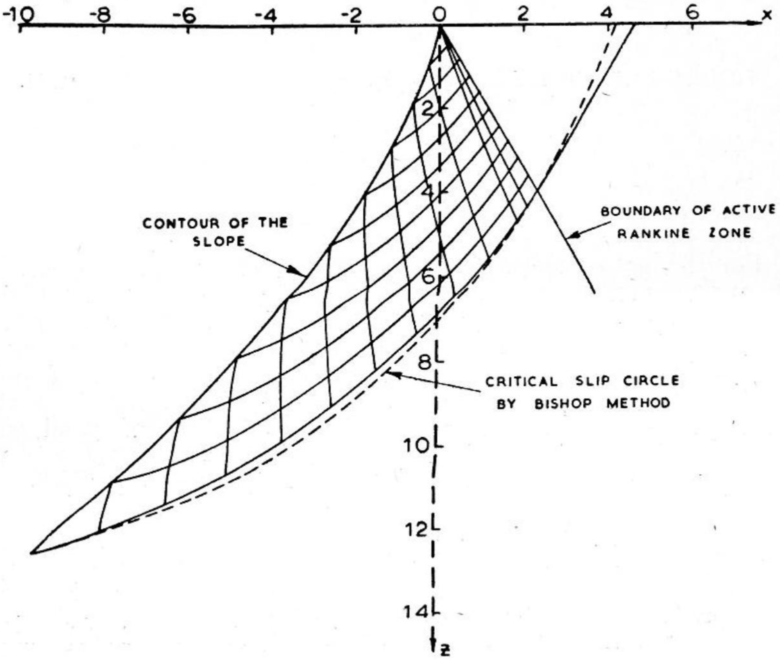

FIGURE 6 : Contour of slope and slip lines for $\phi = 30°$ and $q' = 4{\cdot}4641$.

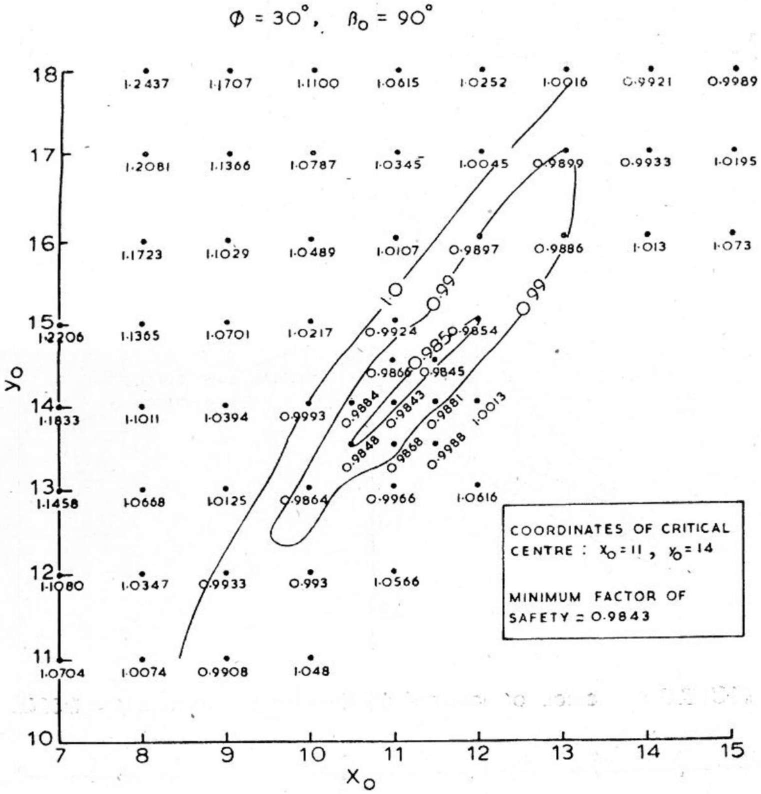

FIGURE 7 : Minimum factor of safety and critical centre for $q' = 3{\cdot}4641$.

For the above slopes (Figures 5 and 6) the minimum factor of safety and the position of the critical centre of the slip circle are obtained by the Bishop's method by trying a number of slip circles, all passing through the toe. The number of slices considered in the zone to the left of z-axis (Figure 5) is the same as the total number of points (excluding the point O), at which the characteristics intersect the contour of the slope. In the zone to the right of z-axis, 5 slices of equal width are considered. The factor of safety obtained in each case along with one or two innermost contours are shown in Figures 7 and 8. It is observed that the minimum factor of safety in the two cases is slightly less than 1, the difference being less than 5 per cent. This indicates that the two approaches show good agreement. The critical slip circles in these two cases as per the Bishop's method are shown on the corresponding slopes in Figures 5 and 6 by dotted lines. These also show good agreement with the rupture surfaces obtained by the method of characteristics.

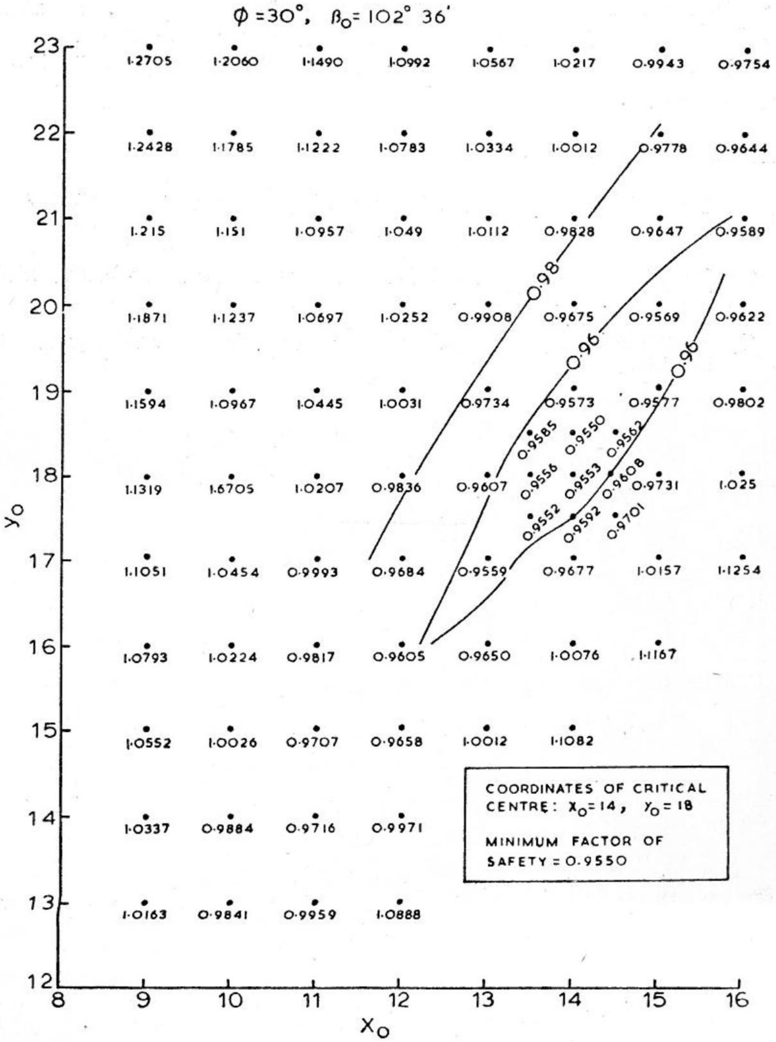

FIGURE 8 : Minimum factor of safety and critical centre for $q' = 4.4641$.

Conclusions

The limited numerical results presented show good agreement between the Bishop's method and the method of characteristics, with regard to both minimum factor of safety and the critical slip surface. For ϕ equal to 30°, for which results are presented herein, lower factor of safety is obtained by the Bishop's method when compared to method of characteristics. In view of the good agreement shown by the two approaches, more detailed numerical work on the above lines will be of great interest.

References

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