Short Communication

Investigations on the Structural Performance of Hyperbolic Paraboloidal Shell Footings on Sand

by

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Introduction

WITH the increasing awareness of the scope of shell structures in foundation engineering, it becomes necessary to investigate the behaviour of these foundations and compare them with conventional ones, so as to establish their relative merits. Among them, individual column footings consisting of hyperbolic paraboloidal shell quadrants, joined together by edge and ridge beams (Figure 1), have come into vogue. These are popularly known as 'umbrella' footings, or more technically as 'hypar' footings.

Various aspects of an extensive investigation conducted by this author, into the general and structural performance, with special emphasis on the aspect of ultimate strength, of reinforced concrete individual square shell footings of the above type, on sand, are being published in various sources ; and as such what has been attempted here is only a general compilation of the more important conclusions that have been obtained as a result of these studies.

Summary and Results

(1) A hyperbolic paraboloidal footing is an economic proposition in situations involving heavy column loads and weak soils. Its economy over the conventional flat footing increases with the increase in the column load, and decreases in the allowable soil pressure, with greater sensitivity to the latter.

(2) Measurement of normal contact pressures on smooth rigid cast iron models of hypar footings, by means of a projecting type of miniature pressure cell, designed and fabricated by this author, revealed that, as in the case of the flat footing, the maximum elastic normal contact pressure occurs at the centre of the shell quadrant, but with values nearly 30 to 40 per cent higher than the average measured. The distribution of contact pressures on the rigid shell and flat models showed close resemblance till the ultimate stage, when the soil failed in bearing.

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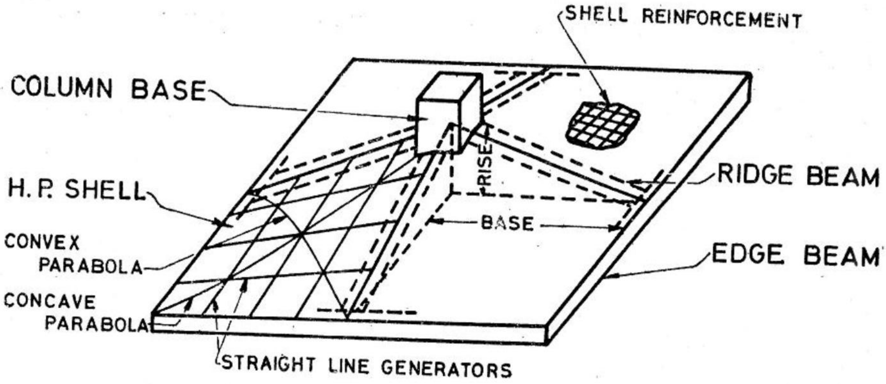

FIGURE 1 : Reinforced concrete individual hypar footing.

Howevere, outdoor tests on reinforced concrete models of hypar footings, indicated a tendency for higher contact pressures on the system of beams and column base, than on the shell proper; the ratio of the averages between the two being nearly 3. This shows that the column load is transmitted to the soil more intensely through the system of beams , thus suggesting the behaviour of a 'fin' footing, where the frame consisting of the beams is primary, with the shells as secondary members serving as fins connecting the members of the frame. This aspect is significant when considering the relative areas between the systems of beams and shells which is much higher in the case of the footing than in the corresponding roof. Another interesting aspect of contact pressure distribution is its tendency for increasing concentration towards the centre with the cracking and progressive failure of the footing.

(3) Theoretical analysis of a case indicated that the maximum Bending Moment in a single hypar shell with rise-to-base ratio of 1/2, on elastic foundation, having simply supported edges and subjected to a central load distributed over a small area, is about 1/3 of that of a similar flat plate. Further, if the above hypar is cut into 4 and joined with beams as an umbrella footing, and if the shells in this footing are taken as a series of beams simply supported at the edges and subjected to a uniformly distributed load, the magnitude of the maximum Bending Moment reduces by 1/11.

(4) Theoretical studies on the ultimate strength of hyperbolic paraboloidal footings show that the contribution of the shell to the ultimate capacity of the footing is compatible with the membrane theory on the working side. As per the author's theory, the ultimate capacity is inversely proportional to the distance of the centre of the soil pressure on a triangular segment, from the centre of the footing, under the given mechanism of collapse.

(5) Tests showed that even though the general structural behaviour of the footing is in accordance with the 'membrane theory' there is considerable bending in the shell and also in the beams, in addition to direct forces. While the direct forces in the shell are such that substantial portions of the shell act in conjunction with the edge beams in the matter of sharing direct forces, the bending is predominantly such as causes tension

at the bottom of the footing. The actual bending action is 'composite' and is not directly obtainable from the bending of the individual shell as by simplified theories.

(6) There is a sudden increase in the stiffness of the hypar footing from a similar ribbed flat plate, even when the former is shallow. This increase is, however, slower with increasing rise of the shell.

(7) Tests on footings (*i*) with edge steel, but no edge beams, and (*ii*) with heavily reinforced edge beams, indicated that edge beams are absolutely essential from the point of view of stiffness of the footing, and also in delaying the onset of cracks. But at the rise-to- base ratio of 1/2 for the shell, which is the normal value used in the field, an increase in the size of the edge beams within a short range over the normal, did not substantially increase the stiffness of the footing or delay cracking.

(8) The shells were found to crack at loads lower than what has been indicated by the membrane theory for the strength of the given concrete. Shrinkage measurements taken in certain cases indicated the possibility of the existence of high initial tensile stresses in the shell, which could cause early cracking.

(9) The behaviour of the footing has been found to be 'composite', in the ultimate stage also. Provision of edge beams considerably bigger in size and reinforcement than required by the membrane theory did not produce enough fixity at the shell edges to enable the shell to fail independently. In other words, the same mechanism of collapse was available over a wide range of edge conditions.

(10) Footings with only the membrane requirement of steel at the centre of the edge beams, where tension is maximum, showed high vulnerability to 'ridge failure'. Provision of extra steel at these locations to the tune of twice the membrane requirement, enabled the footings to partake in a general 'diagonal failure'.

(11) Under the diagonal failure, the ultimate strength of the footing is primarily governed by the yielding of the steel in the shell. The theory developed for the ultimate capacity of the footing, based on the diagonal collapse mechanism is found to be highly satisfactory. The contribution to ultimate strength from heavily reinforced edge beams, can be realized, even partially only by the use of high strength concrete and suitable extra provisions for corner strengthening.

(12) From the point of view of design and construction, as well as elastic and ultimate behaviour the usual rise-to-base ratio of 1/2 for the shell, is found to be satisfactory.

(13) Even though the simple membrane theory serves as a preliminary, but not necessarily conservative, design aid, a sound design of hypar footings must incorporate extra steel at the centre of the edge beams and extra reinforcement at the corner, as well as suitable corner fillets in the footing and inclined fillets in the column base ; besides stipulating the use of a rich mix.

(14) It has been found from tests that provision of reinforcement in the shell (*i*) parallel to the edges, (*ii*) along the diagonals at the same intensity, and (*iii*) along the diagonals at the same intensity in tension, but only at nominal intensity in compression, gave comparable cracking

and collapse loads. This indicates that the real advantage of diagonal steel lies in its scope for reducing the compression steel. From the point of view of grillwork, it appears better to provide the full requirement of steel in the shell, in directions parallel to the edges, with some additional tensile diagonal steel, which besides controlling cracking will contribute to additional strength.

(15) From the results of the present studies, it can be recommended that, the design of the footing may be based on membrane theory, for the requirement of steel. The sections of the shell and edge beam should then be checked by the membrane theory, for tension at the working load. This design should incorporate the above provisions at the critical regions indicated above, which have been identified as a result of the present investigation; and additional reinforcement (optional) in the shell, as described above. Such footings will have a minimum assured load factor equal to (yield stress/allowable stress) of the steel; with a large component of reserve strength in both the 'diagonal' and 'ridge' mechanisms of failure.

(16) Because of its lightness and consequent transportability, and also because of the need for high quality concrete, the hypar footing offers excellent scope for precasting. After controlled production, these precast footings can be installed in the foundation pit ; and irrespective of the soil available at site, dry sand can be poured into the hollow underneath the footing through a hole left in the calumn base. This sand should however be compacted later by interior vibration. However, whether the construction is *in situ* or precast, it is highly important to ensure that there is complete contact between, the soil and the footing, in the interest of preventing excessive deformations and the precipitation of premature failures.

(17) The use of hypar shells in footings shows great promise for extension into rafts, where they can replace the conventional plate elements to great advantage.

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