## Design of Test Bed Foundation for a Helicopter Engine

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## Notations

$A=$ Amplitude in general.
$A_{z}=$ Vertical amplitude.
$A_{x}=$ Horizontal amplitude of translational vibrations.
$A_{\phi}=$ Amplitude of rotational vibrations.
$a_{x}, a_{y}, a_{z}=$ Distances from respective axis.
$C_{Z}=$ Coefficient of uniform compression of soil.
$C_{\phi}=$ Coefficient of non-uniform compression of soil.
$C_{x}=$ Coefficient of uniform displacement of soil.
$F=$ Base area of foundation.
$G_{f}=$ Required weight of foundation.
$I_{x}{ }^{\prime}, I_{y}{ }^{\prime}=$ Moment of inertia of flat foundation.
$I_{2}{ }^{\prime}=$ Polar moment of inertia of flat foundation.
$m=$ Mass in general.
$M=$ Mass of machine.
$N=$ Number of revolutions per minute.
$N_{m}=$ Speed of machine in R.P.M.
$\eta=$ Operating speed of the engine.
$\gamma_{x}, \gamma_{y}=$ Ratio of moments of mass inertia.
$\theta_{x}, \theta_{y}, \theta_{z}=$ Moment of mass inertia related to the axes $x, y$ and $z$, passing through the common centroid of the machine and foundation.
$\theta_{s x}, \theta_{s y}=$ Moment of inertia related to the axes passing through the centre of gravity of the base surface.
$\theta=$ Mass moment of inertia of the equivalent masses about the centre of inertia.
$\lambda=$ Circular frequency of the natural frequency.
$\lambda_{x}=$ Limit frequency of the natural translational vibrations of the foundation.

[^0]$\lambda_{\phi}=$ Limit frequency of the rotational natural vibrations of the foundation.
$\lambda_{1}, \lambda_{2}=$ Principal frequencies.
$\lambda_{\psi}=$ Torsional angular frequency.
$\omega=$ Circular frequency of the generating set.
$\Delta=$ Logrithmic decrement.
$p_{1}, p_{2}=$ Rotational centres of the body.
$s=$ Distance ordinate in general.

## 1. Introduction

THE design of a test bed foundation for a helicopter engine, to be tested after overhauling and before putting it into service, constitutes a specific problem. Although the forces generated by the engine may not be of as high a magnitude as those of a heavy transport plane engine, yet the shape of the engine along with its reductors and the positions of action of the forces are such, that the selection of the size of the foundation block and structural analysis becomes slightly combursome. The paper deals with the foundation design adopted for a particular type of a helicopter engine which forms a part of a Test House Complex.

## 2. Engine Data and Generated Forces

The engine data along with the generated forces was supplied by the manufacturers as given below :-
(a) Horse Power of the Engine

$$
=1700
$$

(b) R.P.M. of the Propeller
$=2600$
(c) Screw Revolutions
$=2000$ R.P.M.
(d) Reductor Revolutions
$=200$ R.P.M.
(e) Weight of the Engine
$=1100 \mathrm{~kg}$.
The generated forces and their points of action are as given in Figure 1.

## 3. Dynamic Soil Characteristics

The school of Research and Training in Earthquacke Engineering, University of Roorkee, Roorkee, was approached to carry out requisite tests to determine the following coefficients :-
(a) The coefficient of elastic uniform compression of soil.
(b) The damping coefficient of soil.

The tests were carried out on a test block foundation of plain cement concrete of size $1.5 \mathrm{~m} . \times 0.75 \mathrm{~m} . \times 0.7 \mathrm{~m}$. high made in a pit of $4.5 \mathrm{~m} . \times 2 \mathrm{~m}$. in plan and of depth $10-15 \mathrm{~cm}$. less than the probable depth of the actual machine foundation. In the present case it was kept as 1.35 m . The pit was made at exactly the same location where the foundation block was to be casted. The school carried out the following tests :-
(a) Cyclic Plate Load Test.
(b) Vertical Resonance Test.
(c) Horizontal Resonance Test

The school recommended the values for the dynamic coefficients of soil as given in Table I.


FIGURE 1 : Generated forces and position of axes.

## 4. Vibration Analysis of the Foundation

### 4.1. SIZE OF THE FOUNDATION BLOCK

The minimum size of the foundation block required to accommodate the engine and its staging was as given in Figure 2. Approximate depth for analysis was calculated by the empirical formula :

$$
G_{f}=K M \sqrt{\eta}
$$

Assuming $K$ as 0.65 for the type of engine

$$
\begin{aligned}
& G_{f}=0.65(2500+2500+1100) \sqrt{2600} \\
& G_{f}=2,02,000 \mathrm{~kg} .
\end{aligned}
$$

TABLE I
Coefficient of Uniform Compression
( $C_{Z}$ ).

| Type of Test | $C_{Z}$ for the area of the <br> Block $\left(\mathrm{kg} . / \mathrm{cm}^{3}.\right)$ | $C_{Z}$ for $10 \mathrm{~m}^{2}$. <br> area (kg./cm $\left.{ }^{3}\right)$ | Damping factor <br> $(\%)$ |
| :---: | :---: | :---: | :---: |
| Vertical Resonance <br> Test | 4.17 | 1.40 | 11.51 |
| Horizontal Resonance <br> Test | 578 | 1.94 | - |

Recommended value of $C_{Z}$ for design : $1.7 \mathrm{~kg} . / \mathrm{cm}^{3}$. for $10 \mathrm{~m}^{2}$. area.


FIGURE 2: Plan showing placement of engine on to the Foundation.

$$
\text { Depth } \begin{aligned}
(d) & =\frac{2,02,000}{(\text { Area) } 2400} \\
& =\frac{2,02,000}{[7(6.5)+5(3)] 2400} \\
& =1.41 \mathrm{~m} .
\end{aligned}
$$

The depth of the foundation block for design and analysis has been taken as 1.60 m

### 4.2. DETERMINATION OF THE C.G. OF THE SYSTEM

With reference to Figures 1 and 2, and considering static loads only, the $C . G$. with respect to $X, Y$, and $Z$ axis has been worked out in Tables II and III.
4.3. MASS MOMENT OF INERTIA ABOUT THE AXES THROUGH THE C.G.

### 4.3.1. About $X$-axis

(i) Machine $\left(M_{1}\right)=\frac{m}{12}\left(a_{y}{ }^{2}+a_{z}{ }^{2}\right)+m r_{x}{ }^{2}{ }^{(1)}$

$$
\begin{aligned}
& =\frac{2 \cdot 50}{9 \cdot 81(12)}(\text { Negligible })+\frac{2 \cdot 50}{9 \cdot 81}(4 \cdot 185-0 \cdot 86)^{2} \\
& =2 \cdot 8 \mathrm{tm} . \mathrm{sec}^{2}
\end{aligned}
$$

(ii) Reductor $\left(M_{2}\right)=\frac{m}{12}\left(\frac{3}{4} D^{2}+1^{2}\right)+m r_{x}^{2} \quad$ (For circular section)

$$
\begin{array}{r}
=\frac{1 \cdot 10}{9 \cdot 81(12)}\left[\frac{3}{4}(1 \cdot 75)^{2}+1 \cdot 35^{2}\right]+ \\
\frac{2 \cdot 50}{9 \cdot 81}(3.05-0.86)^{2}
\end{array}
$$

$$
=1.22 \mathrm{tm} . \mathrm{sec}^{2}
$$

(iii) Machine $\left(M_{3}\right)=\frac{2 \cdot 50}{9 \cdot 81(12)}\left(2^{2}+0 \cdot \epsilon 0^{2}\right)+\frac{2 \cdot 50}{9 \cdot 81}(2 \cdot 14-0 \cdot 86)^{2}$

TABLE II
$X$ (Moments about $O Z^{\prime}$ )

| Sl. No. | Load in kg. | Lever arm in <br> metres | Moments <br> kg.m. | Remarks |
| :--- | ---: | ---: | ---: | :--- |
| 1. | 2,500 | 4.09 | 10,230 | Machine Load |
| 2. | 1,100 | 7.09 | 7,800 | Reductor Load |
| 3. | 2,500 | 9.09 | 22,700 | Machine Load |
| 4. | $2,32,000$ | 4.98 | $11,51,000$ | Foundation load acting <br> at its own C.G. |
| Sum | $2,38,100$ |  |  |  |

$$
\bar{X}=\frac{11,91,730}{2,38,100}=5 \mathrm{~m}
$$

Eccentricity $=5.00-4.98=0.02 \mathrm{~m}$. or 20 mm .
Permissible eccentricity $=5$ per cent of shorter length $=335 \mathrm{~mm}$.

TABLE III
$\dddot{Z} \bar{Y}$ (Moments about $O-X^{\prime}$ )

| Sl. No. | Load in kg. | Lever arm in <br> metres | Moments in <br> kg. m. | Remarks |
| :--- | :---: | :---: | :---: | :---: |
| 1. | 2,500 | 4.185 | 10,460 | Machine Load |
| 2. | 1,100 | 3.05 | 3,355 | Reductor Load |
| 3. | 2,500 | 2.14 | 5,350 | Machine Load |
| 4. | $\frac{2,32,000}{2,38,100}$ | 0.80 | $\frac{1,85,600}{2,04,765}$ | Foundation Load |
| Sum | $\bar{Z}=\frac{2,04,765}{2,38,100}=0.86 \mathrm{~m}$. |  |  |  |

Eccentricity $=0.86-0.80=0.06 \mathrm{~m}$. or 60 mm .
Permissible eccentricity $=5$ per cent of depth $=80 \mathrm{~mm}$.

$$
Y=3.25 \mathrm{~m}
$$

$$
\begin{aligned}
& =0.0925+0.416 \\
& =0.5085 \mathrm{tm} . \mathrm{sec}^{2}
\end{aligned}
$$

(iv) Foundation $=-\frac{174.4}{9.81(12)}\left(6.5^{2}+1.6^{2}\right)+\frac{57.6}{9.81 \times 12}\left(3^{2}+1.6^{2}\right)$

$$
\begin{aligned}
& +\frac{232}{9.81}(0.86-0.80)^{2} \\
= & 70.233 \mathrm{tm} \cdot \mathrm{sec}^{2} \\
\theta_{x}(\text { Sum })= & 2.80+1.22+0.5085+70.233 \\
= & 74.76 \mathrm{tm} \cdot \mathrm{sec}^{2}
\end{aligned}
$$

### 4.3.2. About Y-axis

(i) Machine $\left(M_{1}\right)=\frac{m}{12}\left(a_{x}{ }^{2}+a_{z}{ }^{2}\right)+m r_{y}{ }^{2}$

$$
=\text { Negligible item }+\frac{2.50}{9.81}\left(3.325^{2}+1.91^{2}\right)
$$

$=3.72 \mathrm{tm} . \mathrm{sec}^{2}$.
(ii) Reductor $\left.\left(M_{2}\right)=\frac{m D^{2}}{8}+m r_{y}{ }^{2( }{ }^{1}\right) \quad$ (For the circular section).

$$
\begin{aligned}
& =\frac{1.10}{9.81(8)}\left(1.75^{2}\right)+\frac{1.10}{9.81}\left(2.09^{2}+2.19^{2}\right) \\
& =1.065 \mathrm{tm} . \mathrm{sec}^{2} .
\end{aligned}
$$

(iii) Machine $\left(M_{3}\right)=\frac{m}{12}\left(a_{x}{ }^{2}+a_{z}{ }^{2}\right)+m r_{y}{ }^{2}$

$$
\begin{aligned}
& =\frac{2.50}{9.81(12)}\left(2^{2}+0.60^{2}\right)+\frac{2.50}{9.81}\left(4.09^{2}+1.28^{2}\right) \\
& =4.7325 \mathrm{tm} . \mathrm{sec} .^{2}
\end{aligned}
$$

(iv) Foundation $=\frac{174.4}{9.81(12)}\left(7^{2}+1.6^{2}\right)+\frac{57.6}{9.81(12)}\left(5^{2}+1.6^{2}\right)$

$$
\begin{aligned}
& +\frac{232}{9.81}\left(0.02^{2}+0.06^{2}\right) \\
& =76.40+13.45+0.0945 \\
& =89.945 \mathrm{tm} . \mathrm{sec}^{2} . \\
\theta_{y} & =3.720+1.065+4.733+89.945 \\
\theta_{y} & =99.463 \mathrm{tm}, \mathrm{sec}^{2},
\end{aligned}
$$

### 4.3.3. About Z-axis

(i) Machine $\left(M_{1}\right)=\frac{m}{12}\left(a_{x}{ }^{2}+a_{y}{ }^{2}\right)+m r_{z}{ }^{2}$
$=$ Negligible $+\frac{2.50}{9.81}(5-4.09)^{2}$
$=0.21 \mathrm{tm} . \mathrm{sec}^{2}$.
(ii) Machine $\left(M_{2}\right)=\frac{m}{12}\left(\frac{3}{4} D^{2}+1^{2}\right)+m r_{z}{ }^{2} \quad$-(Being a circular section).

$$
\begin{aligned}
& =\frac{1.10}{9.81(12)}\left(\frac{3}{4} 1.75^{2}+1.35^{2}\right)+\frac{1.10}{9.81}(2.09)^{2} \\
& =0.486 \mathrm{tm} . \mathrm{sec}^{2}
\end{aligned}
$$

(iii) Machine $\left(M_{3}\right)=\frac{2.50}{9.81(12)}\left(2^{2}+2^{2}\right)+\frac{2.50}{9.81}(4.09)^{2}$

$$
=0.167+4.225
$$

$$
=4.392 \mathrm{tm} . \mathrm{sec}^{2}
$$

(iv) Foundation $\quad=\frac{174.40}{9.81(12)}\left(7^{2}+6.5^{2}\right)+\frac{57.6}{9.81(12)}\left(5^{2}+3^{2}\right)$

$$
\begin{aligned}
& +\frac{132}{9.81}(0.02)^{2} \\
= & 151.259 \mathrm{tm} \cdot \mathrm{sec}^{2} \\
\theta_{Z}(\text { Sum }) & =0.21+0.486+4.392+151.259 \\
= & 156.347 \mathrm{im} \cdot \mathrm{sec}^{2} .
\end{aligned}
$$

### 4.4. MASS MOMENT OF INERTIA ABOUT THE C.G. OF THE BASE

$$
\begin{aligned}
\text { Total mass }(m) & =\frac{\operatorname{Sum} G}{g} \\
& =\frac{2.50+1.10+2.50+232}{9.81} \\
& =\frac{238.10}{9.81} \\
m & =24.30 \mathrm{t} \mathrm{sec}^{2} . / \mathrm{m} . \\
\theta_{s x} & =\theta_{x}+m \bar{X}^{2} \\
& =74.76+24.30(0.86)^{2} \\
\theta_{s x} & =92.76 \mathrm{tm}^{2} \mathrm{sec}^{2} .
\end{aligned}
$$

$$
\begin{aligned}
\theta_{s y} & =\theta_{y}+m \bar{Z}^{2} \\
& =99.463+24.30(0.86)^{2} \\
\theta_{s y} & =117.463 \mathrm{tm} . \mathrm{sec}^{2} . \\
\theta_{s z} & =\theta_{z}=156.347 \mathrm{tm} . \mathrm{sec}^{2}
\end{aligned}
$$

The ratios of mass moment of inertia are :

$$
\begin{aligned}
& r_{x}=\frac{\theta_{x}}{\theta_{s x}}=\frac{74.76}{92.76}=0.806 \\
& r_{y}=\frac{\theta_{y}}{\theta_{s y}}=\frac{99.463}{117.463}=0.93
\end{aligned}
$$

4.5. BASE AREA AND MOMENT OF INERTIA OF THE FOUNDATION BLOCK

Base area

$$
\begin{aligned}
& =7(6.5)+5(3) \\
F & =60.50 \text { sq. } \mathrm{m} . \\
I_{x}^{\prime} & =\frac{7(6.5)^{3}}{12}+\frac{5(3)^{3}}{12} \\
I_{x}^{\prime} & =171.25 \mathrm{~m}^{4} . \\
I_{y}^{\prime} & =\frac{6.5(7)^{3}}{12}+\frac{3(5)^{3}}{12}+36(1.02)^{2}+24.5(1.48)^{2} \\
I_{y}^{\prime} & =623.20 \mathrm{~m}^{4} .
\end{aligned}
$$

The polar moment of inertia :

$$
\begin{aligned}
& I_{Z}^{\prime}=I_{y}^{\prime}+I_{x}^{\prime}=623.20+171.25 \\
& I_{Z}^{\prime}=7.94 .45 \mathrm{~m}^{4}
\end{aligned}
$$

### 4.6. DYNAMIC SOIL COEFFICIENT USED IN THE ANALYSIS

The value of the coefficient of uniform elastic compression for $10 \mathrm{~m}^{2}$. as recommended by the school of Earthquake Engineering $=1.7 \mathrm{~kg} . / \mathrm{cm}^{3}$. The relationship used to determine the value of $C_{Z}$ for the area of $60.50 \mathrm{~m}^{2}$. is

$$
\begin{aligned}
\frac{C_{Z_{1}}}{C_{Z_{2}}} & =\left(\frac{A_{2}}{A_{1}}\right)^{0.35\left(^{3}\right)} \\
C_{Z} & =0.9 \mathrm{~kg} . / \mathrm{cm}^{3} . \text { or } 900 \mathrm{t} . / \mathrm{m}^{3} . \\
C_{x} & =\frac{Z}{2}=0.45 \mathrm{~kg} . / \mathrm{cm}^{3} . \text { or } 450 \mathrm{t} . / \mathrm{m}^{3} \\
C_{\phi} & =2 C_{Z}=1.80 \mathrm{~kg} . / \mathrm{cm}^{3} . \text { or } 1,800 \mathrm{t} . / \mathrm{m}^{3} . \\
C_{\psi} & =\frac{C_{Z}}{2}=0.45 \mathrm{~kg} . / \mathrm{cm}^{3} . \text { or } 450 \mathrm{t} . / \mathrm{m}^{3}
\end{aligned}
$$

### 4.7. THE DETERMINATION OF NATURAL VIBRATIONS IN $X-Z$ PLANE

4.7.1. The square of the limit frequency of the rotational natural vibrations in the $X-Z$ plane is according to the equation :-

$$
\begin{aligned}
\lambda_{\phi}^{2} & =\frac{C_{\phi} I_{v}^{\prime}}{\theta_{s y}}=\frac{1800(623 \cdot 20)}{117 \cdot 463} \\
& =9,550 \mathrm{sec} .^{-2}
\end{aligned}
$$

The square of the limit frequency of the natural translational vibration is calculated by the equation :-

$$
\begin{aligned}
\lambda_{x}^{2} & =\frac{C_{x} F}{m}=\frac{450(60 \cdot 50)}{24 \cdot 30} \\
\lambda_{x}^{2} & =1120 \mathrm{sec} .^{-2}
\end{aligned}
$$

4.7.2. The fundamental frequencies occurring in the $X-Z$ plane can be obtained by solving the following equation of the second degree for $\lambda^{2}$ :-

$$
\begin{aligned}
& \lambda^{4}-\frac{\lambda_{\phi}^{2}+\lambda_{x}^{2}}{r_{v}} \lambda^{2}+\frac{\lambda_{\phi}^{2} \lambda_{x}^{2}}{r_{y}}=0 \\
& \lambda^{4}-\frac{9550+1120}{0.93} \lambda^{2}+\frac{9550(1120)}{0.93}=0 \\
& \lambda_{1}{ }^{2}=1.0345\left(10^{4}\right) \text { or } \lambda_{1}=101.5 \mathrm{sec} .^{-1} \\
& \lambda_{2}{ }^{2}=0.1105\left(10^{4}\right) \text { or } \lambda_{2}=33.2 \mathrm{sec} .^{-1}
\end{aligned}
$$

4.7.3. The natural frequencies of the foundation block can be obtained from the fundamental frequencies and

$$
\begin{aligned}
& N_{1}=\frac{60}{2}(101 \cdot 5)=967 \text { C.P.M. } \\
& N_{2}=\frac{60}{2}(33 \cdot 2)=316 \text { C.P.M. }
\end{aligned}
$$

Both these natural frequencies are well below the speed of the engine (2600 R.P.M.) and therefore, resonance cannot occur.

### 4.8. THE DETERMINATION OF NATURAL VIBRATIONS IN Y-Z PLANE

 4.8.1. The square of the limit frequency of rotational vibration$$
\begin{aligned}
\lambda_{\phi}^{2} & =\frac{C_{\phi} I_{x}^{\prime}}{\theta_{s x}}=\frac{1800(171 \cdot 25)}{92 \cdot 76} \\
\lambda_{\phi}^{2} & =3,320 \mathrm{sec} .^{-2}
\end{aligned}
$$

The square of the limit frequency of translational vibration

$$
\begin{aligned}
\lambda_{x}^{2} & =\frac{C_{x} F}{m}=\frac{450(60 \cdot 50)}{24 \cdot 30} \\
\lambda_{x}^{2} & =1120 \mathrm{sec} \cdot{ }^{-2}
\end{aligned}
$$

4.8.2. The fundamental frequencies are obtained as before

$$
\begin{gathered}
\lambda^{4}-\frac{\lambda_{\phi}^{2}+\lambda_{x}^{2}}{\gamma_{x}} \lambda^{2}+\frac{\lambda_{\phi}^{2} \lambda_{x}^{2}}{\gamma_{x}}=0 \\
\lambda^{4}-\frac{3320+1120}{0.806} \lambda^{2}+\frac{3320(1120)}{0.806}=0 \\
2=0.448\left(10^{4}\right) \text { or } \lambda_{1}=67 \mathrm{sec} .^{-1} \\
1 \\
\lambda_{2}^{2}=0.104\left(10^{4}\right) \text { or } \lambda_{2}=32.25 \mathrm{sec} .^{-1}
\end{gathered}
$$

4.8.3. Therefore, the natural frequencies of the foundation block in the $Y-Z$ plane are :

$$
\begin{aligned}
& N_{1}=\frac{67}{0 \cdot 105}=640 \text { C.P.M. } \\
& N_{\mathbf{3}}=\frac{32 \cdot 25}{0 \cdot 105}=308 \text { C.P.M. }
\end{aligned}
$$

### 4.9. THE ROTATIONAL NATURAL VIBRATION ABOUT THE Z-AXIS

This can be calculated by the relationship :-

$$
\begin{aligned}
\lambda_{\psi} & =\frac{450(794 \cdot 45)}{156 \cdot 347}=47.7 \mathrm{sec} .^{-1} \\
N_{\psi} & =\frac{47 \cdot 7}{0.105}=455 \text { C.P.M. }<2600 \text { R.P.M. }
\end{aligned}
$$

4.10. DETERMINATION OF THE AMPLITUDES
4.10.1. The Amplitude due to Vertical Eccentric Force: The vertical generated force of 2.0 tonne acting at a distance of 0.91 m . from the centroid, is converted to a equivalent force of 2 tonne and a moment of $2 \times 0.91=1.82 \mathrm{t} . \mathrm{m}$. acting at the centroid of the combined masses of machine and foundation block.
(a) Amplitude due to the vertical force $2 t\left(K_{z}\right)$ passing through the centroid can be calculated by the relationship :

$$
\begin{aligned}
& A_{z}=\frac{K_{z}}{m\left(\lambda_{z}^{2}-\omega^{2}\right)} \\
&\left.{ }^{1}\right) \\
& \lambda_{z}^{2}=\frac{C_{z} F}{m} \\
& \lambda_{z}^{2}=\frac{900(60 \cdot 5)}{24 \cdot 30}=2240 \mathrm{sec}^{-2} . \\
& \omega^{2}=(2600) 0 \cdot 105^{2}=74,000 \mathrm{sec}^{-2} . \\
& A_{z}=\frac{2}{24 \cdot 30(2240-74,000)} \\
& A_{z}=-1 \cdot 15\left(10^{6}\right) \mathrm{m} . \\
& A_{z}=-0 \cdot 00115 \mathrm{~mm} .<0 \cdot 20 \mathrm{~mm} .
\end{aligned}
$$

The sign of the amplitude is of no consequence.
(b) Due to the generated moment of 1.82 t . m., the displacement of centre of rotation can be calculated by the formulae :

$$
\begin{aligned}
& A_{1}=\frac{M}{m p_{2}\left(\lambda_{2}{ }^{2}-\omega^{2}\right)} \\
& A_{2}=\frac{M}{m p_{1}\left(\lambda_{1}{ }^{2}-\omega^{2}\right)}
\end{aligned}
$$

The rotational centres of the body ( $p_{1}$ and $p_{\mathbf{2}}$ ) are calculated from the limit and angular frequencies :

$$
\begin{aligned}
& p_{4}=\frac{\lambda_{x}^{2} s}{\lambda_{x}^{2}-\lambda_{1}^{2}} \\
& p_{2}=\frac{\lambda_{x}^{2} s}{\lambda_{x}^{2}-\lambda_{z}^{2}}
\end{aligned}
$$

Substituting the values from 4.7.1 and 4.7.2

$$
p_{1}=\frac{1120(5 \cdot 25)}{1120-10345}=-0.635 \mathrm{~m} .
$$

$p_{1}$ being negative, the centre of rotation lies above the centroit $t, e$ displacement and rotation are of opposite sign

$$
p_{2}=\frac{1120(5 \cdot 24)}{1120-1105}=+391 \mathrm{~m} .
$$

In this case as ${ }_{x}^{2} \bumpeq \lambda_{z}^{2}$ this will cause on'y displacement and no rotation would occur. Substituting the values

$$
\begin{aligned}
A_{2} & =\frac{M}{m p\left(\lambda_{1}^{2}-\omega^{2}\right)} \\
& =\frac{1 \cdot 82}{24 \cdot 3(10,345-74,000)(-0.635)} \\
& =0.183\left(10^{-5}\right) \mathrm{m} . \\
& =0.00183 \mathrm{~mm} .
\end{aligned}
$$

The amount of displacement is quite negligible.
4.10.2. Rotational displacement due to moment about vertical axis (Z-axis) of the magnitude of 6.33 tm is calculated by the following relationship :

$$
\begin{aligned}
A_{\psi} & =\frac{M_{z}}{0_{z}\left(\lambda_{\psi}^{2}-\omega^{2}\right)} \\
& =\frac{6.33}{156.347\left(47.7^{2}-74,000\right)} \\
& =0.564\left(10^{-6}\right)
\end{aligned}
$$

Displacement at top of staging $=0.564\left(10^{-6}\right)(5.24)$

$$
=2 \cdot 95\left(10^{-6}\right) \mathrm{m}
$$

Maximum displacement $\quad=0.00295 \mathrm{~mm}$.
The displacements caused due to generated forces being quite negligible and very much on safer side and therefore the effect of the reductor torque of $609 \mathrm{~kg} . \mathrm{m}$. has not been considered in the design.

## 5. Structural Design of the Foundation Block

### 5.1. DETERMINATION OF SOIL STRESSES

5.1.1. Stresses due to static loads with respect to the $X$-axis are :

$$
\begin{aligned}
\sigma & =\frac{W}{A} \pm \frac{B M}{Z} \\
\text { Eccentricity } & =0.02 \mathrm{~m} . \\
Z & =\frac{I_{y}^{\prime}}{4.98}=\frac{623.2}{4.98} \\
Z & =125 \\
\frac{\sigma_{A}}{\sigma_{B}} & =\frac{238.10}{60.5} \pm \frac{238.10(0.02)}{125} \\
& =3.94 \pm 0.038 \\
\sigma_{A} & =3.978 \mathrm{t} . / \mathrm{m}^{2} \\
\sigma_{B} & =3.902 \mathrm{t} . / \mathrm{m}^{2}
\end{aligned}
$$

The effect of the eccentricity is quite negligible.
5.1.2. Stresses due to Eccentric Vertical Dynamic Force. As is evident from 4.10.1 (b) that there is going to be no rotation but only displacement will occur, therefore

$$
V_{d}=\mu C_{Z} F A_{z}
$$

Assuming

$$
\begin{aligned}
\mu & =3 \\
V_{d} & =3(900)(60 \cdot 5)(-1 \cdot 15)\left(10^{-6}\right) \\
V_{d} & =-0.188 \mathrm{t}
\end{aligned}
$$

This reaction will be acting in the opposite direction to the soil stresses introduced due to the vertical static loads, and magnitude being small, this force has not been considered for finding out the maximum stresses.
5.1.3 The dynamic moment introduced due to the moment about the vertical axis ( $Z$-axis) can be found out by the following equation :

$$
M_{Z d}=\mu C_{\psi} I_{z}^{\prime} A_{\psi}
$$

Taking the fatigue factor $\mu$ as 3

$$
\begin{aligned}
& M_{Z d}=3(450)(794.45)(0.564)\left(10^{-6}\right) \\
& M_{Z d}=0.604 \mathrm{t} \mathrm{~m}
\end{aligned}
$$

Stresses introduced due to this dynamic moment :

$$
\sigma= \pm \frac{0.604}{125}=000484 \mathrm{t} . / \mathrm{m}^{2} . \text { say } 0.005 \mathrm{t} . / \mathrm{m}^{2}
$$

Therefore overall soil stresses :

$$
\begin{aligned}
& \sigma_{A}=3.978 \pm 0.005=3.983 \mathrm{t} . / \mathrm{m}^{2} . \text { or } 3.973 \mathrm{t} . / \mathrm{m}^{2} \\
& \sigma_{B}=3.902 \pm 0.005+3.906 \mathrm{t} . / \mathrm{m}^{2}, \text { or } 3.897 \mathrm{t} . / \mathrm{m}^{2}
\end{aligned}
$$

The safe permissible bearing capacity of soil being $8 \mathrm{t} . / \mathrm{m}^{2}$., the soil stresses introduced are within safe limits.

### 5.2. DETERMINATION OF THE STRESSES IN THE FOUNDATION BLOCK

5.2.1. The net ground reactions after taking into account the self weight of the foundation block, works out to be very nominal as is evident from Figure 3. Assuming the ground reaction to be uniformly distributed of the higher magnitude, i.e., $116 \mathrm{~kg} . / \mathrm{m}$. the bending moments introduced at different sections are as follows :

$$
\begin{aligned}
B . M I & =\frac{116(\mathrm{I} \cdot 4)^{2}}{2}=114 \mathrm{~kg} \cdot \mathrm{~m} \\
B . M I I & =\frac{116(4 \cdot 2)^{2}}{8}-114 \\
& =255-114=141 \mathrm{~kg} . \mathrm{m}
\end{aligned}
$$


(i) SELF WEIGHT OF THE FOUNDATION BLOCK


NET GROUND REACTION FIG.(i) + FIG.(ii)
FIGURE 3 : Ground reaction and superimposed loads.


FIGURE 4: Part section at BB showing reinforcement detail.
Maximum bending moments introduced in the other direction ( $0-Y$ )

$$
=\frac{116(5)^{2}}{8}=62 \mathrm{~kg} . \mathrm{m}
$$

5.2.2. Reinforcements required: The bending moments introduced in

the different sections of the foundation block are very nominal. Taking the following details for design :
(a) Mix: M 150 grade as per I.S. 456-1964
(b) Steel: Mild steel plane bar

$$
\sigma_{s v}=2600 \mathrm{~kg} . / \mathrm{cm}^{2} .
$$

(c) Depth of foundation $=160 \mathrm{~cm}$.
(d) Effective depth $=155 \mathrm{~cm}$.

$$
R=\frac{36200}{100(155)^{2}}=0.015
$$

## 6. Working Details of the Foundation Block

6.1. Minimum reinforcement provided is 0.1 per cent of the crosssection area

$$
=\frac{0 \cdot 1}{100}(100)(160)=16 \mathrm{~cm}^{2} . \text { or } 20 \mathrm{~mm} .
$$

diameter bars have been provided @ $20 \mathrm{~cm} . C / C$ in shorter direction and 16 mm . diameter bars in the other direction as shown in the details of Figures 4 and 5.
6.2. Figure 4 shows the provision of ISMB 300 at top of the foundation block. These have been provided to facilitate the fixing of the staging of the helicopter engine, through plates and bolts tightened with these beams. The beams have been welded with an arrangement of 16 mm . bars (refer Figure 4) in such a way that these should behave monolithically with the main foundation block.
6.3. Figures 6 and 7 show the details of the steel guards which have been designed in such a manner so that the top beams could be placed at


FIGURE 7: Section showing connection of beams with levelling stands.
exact desired levels with geodetic accuracy. Prime importance has been given to the levels as even a slight difference of level would affect the overall functioning of the foundation block and therefore the testing of the engine.
6.4. The foundation block is separated from the main building foundation by providing a 100 mm . thick gap filled with a workable mixture of saw dust and bitumen. This has been provided to give an additional safety against transference of any vibration to the building whatsoever (refer Figure 4).

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