A New Foundation for Tower-Shaped Structures

by

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NOTATIONS

N_ϕ, N_θ and $N_{\theta\phi}$ = Meridional force, hoop force and shearing force per unit length of shell, respectively.

a = Radius of spherical shell.

p_ϕ, p_θ and p_r = Load components along tangent to meridian, parallel circle and along radial direction per unit area of shell surface, respectively.

A_n ; B_n ; A_1 ; B_1 = Arbitrary constants.

p_{ϕ_1}, p_{θ_1} and p_{r_1} = Load components corresponding to first harmonic.

p = Pressure due to soil reaction.

ϕ_o = Angle subtended at the free end of shell.

ϕ_c = Angle subtended at the fixed end of shall.

$\overline{N}_\phi$ and $\overline{N}_\theta$ = Non-dimensional form of N_ϕ and N_θ corresponding to symmetrical state of loading.

$\overset{*}{N}_\phi$, $\overset{*}{N}_\theta$ and $\overset{*}{N}_{\theta\phi}$ = Non-dimensional form of N_ϕ, N_θ and $N_{\theta\phi}$ corresponding to anti-symmetrical state of loading.

h = Maximum rise of shell.

D = Diameter of shell in plan.

${}^SN_{\phi,\,\max}$; ${}^SN_{\theta,\,\max}$ = Maximum value of stress resultants corresponding to symmetrical loading.

${}^AN_{\phi,\,\max}$; ${}^AN_{\theta,\,\max}$ = Maximum value of stress resultants corresponding to anti-symmetrical loading.

${}^CN_{\phi,\,\max}$; ${}^CN_{\theta,\,\max}$ = Maximum value of stress resultant after superimposing for both cases of loading.

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Introduction

THE load is transmitted to the foundation of tall circular structures like water-towers, chimney stacks, cooling towers, *etc.*, by means of either columns or through shell structures (Figure 1). It is customary to use a raft slab for transmitting the load finally to the soil. However, it is to be noted that as the height of such structures increases the size of the raft slab increases abnormally. It has been found that if a raft foundation is replaced by a shell foundation, economy as much as 50 per cent can be achieved ([1]).

As the mathematical analyses for roof shells are easily available, the same analyses can be extended to the foundation shells also. Foundation shells possess a number of points in which they differ from the shells used for roofing.

With regard to the high stresses acting on the foundations, the ratio of thickness to span is higher than that of the roof shells. Foundation shells are supported on the sub-soil by their whole surface and are subjected to both the reactive resistance of the soil and to the action due to earth layers above the foundation. The above mentioned factors reduce the danger of stability failure of foundation shells and as such consideration of stability failure in design may be neglected. The foundation shells, resting by their whole surface on the soil, do not require expensive shuttering for their construction because it is the duly excavated and shaped soil surface which serves as formwork for the concreting of the shell.

Both single-curvature and double-curvature shells are used for foundations (Figure 2). The present study is restricted to the use of

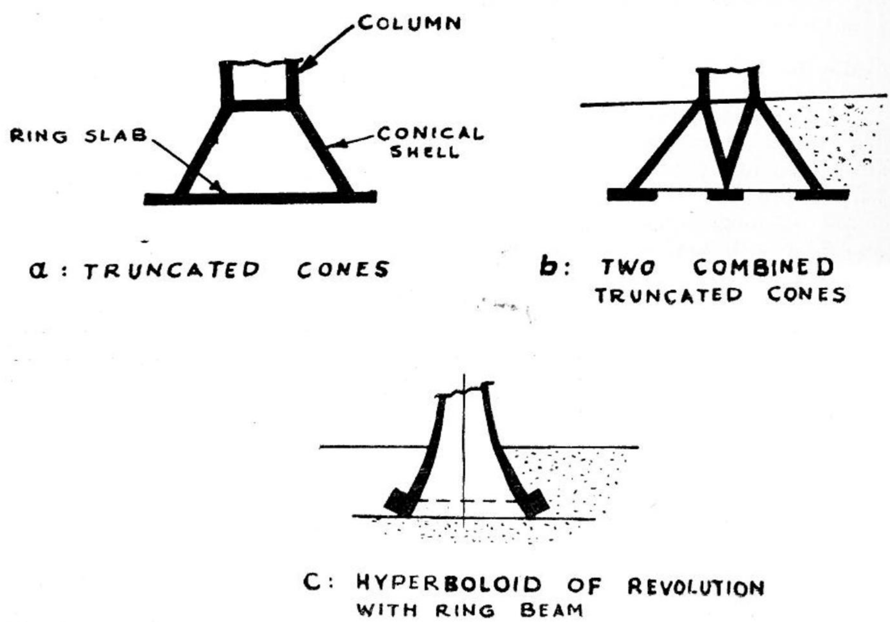
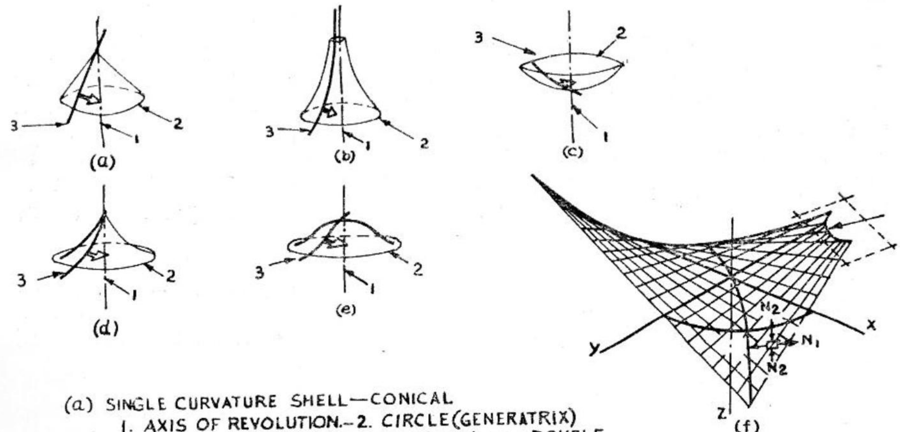


FIGURE 1: Foundation shells.



(a) SINGLE CURVATURE SHELL—CONICAL
1. AXIS OF REVOLUTION.–2. CIRCLE(GENERATRIX)
3. STRAIGHT LINE AS DIRECTRIX, b, c, d, e, e– DOUBLE CURVATURE SHELL

(b) HYPERBOLICAL

(c) SPHERICAL

(d-e) PARABOLICAL

(f) HYPERBOLICAL PARABOLOID

FIGURE 2 : Geometry of shells used as foundations for tower-shaped structures.

domical shells as foundation. This type of shell can have its convexity either in upward or downward direction. It is evident that shells having cavity facing upwards would distribute the pressure more uniformly in the sub-soil. This has been confirmed by photo-elastic investigation performed in Hungary(1) on models of foundations of different surface shape, Figure 3.

Analysis

The dead load coming from the super-structure of a tower acts symmetrically over the footing giving rise to a uniform pressure from the soil. Whereas the wind forces acting on the super-structure give rise to varying pressure from the soil, hence the spherical shell has been analysed, based on membrane analysis for both types of loading, *i.e.*, symmetrical as well as anti-symmetrical snow loading (Figure 4-I).

The stress resultants (membrane forces) with positive loading components acting on the surface of the shell, acting on a typical shell element are shown in Figure 5. Referring to Figure 5, the equations of equilibrium can be expressed as

$$\partial N_{\theta\phi}/\partial\theta + \sin\phi\frac{\partial N_\phi}{\partial\phi} + N_\phi \cos\phi - N_\theta \cos\phi + p_\phi\, a \sin\phi = 0 \quad ..(1)$$

$$\sin\phi\,\frac{\partial N_{\theta\phi}}{\partial\phi} + \frac{\partial N_\theta}{\partial\theta} + 2\cos\phi\, N_{\theta\phi} + p_\theta\, a \sin\phi = 0 \quad ..(2)$$

$$N_\theta + N_\phi = a\, p_r \,..(3)$$

On solving Equations (1) to (3) spherical shell subjected to either symmetrical or anti-symmetrical loading can be analysed.

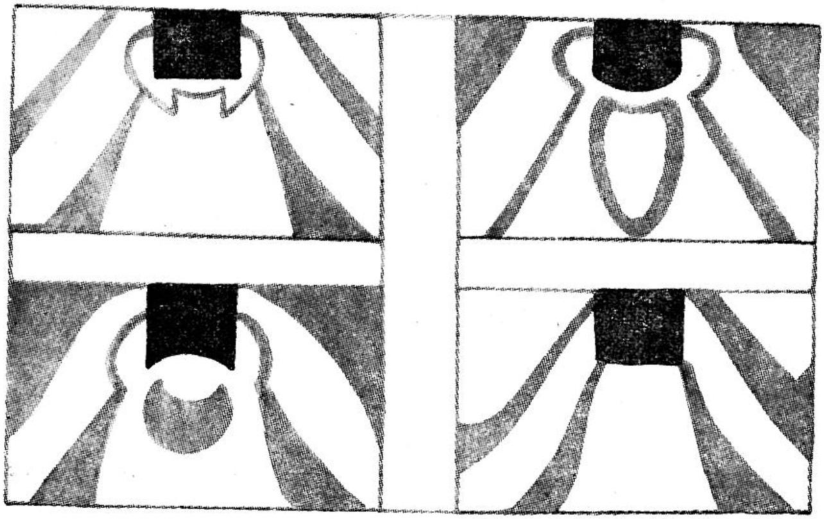


FIGURE 3 : Results of photo-elastic tests of different shapes of foundation base of structures.

(A) SYMMETRICAL CASE

By eliminating θ terms and solving Equations (1) and (2), Flügge expresses the meridional force (N_ϕ) as :

$$N_\phi = \frac{1}{a \sin^2 \phi}\left[\int a^2 (p_r \cos \phi - p_\phi \sin \phi) \sin \phi \, d_\phi + C\right] \quad ..(4)$$

Where C represents the effect of loads which may be applied above the circle $\phi = \phi_0$. In the present case, value of C is zero (Refer Figure 4-I). Now referring to Figures 4-I and II the loading components may be expressed as

$$p_\phi = p \cos \phi \sin \phi$$

$$p_r = -p \cos^2 \phi$$

$$p_\theta = 0 \quad ..(5)$$

Substituting these loading components and applying limits $\phi = (180 - \phi_0) = \phi'$ (say) to $\phi = (180 - \phi) = \phi''$ $(\phi' < \phi'')$; where ϕ'' is any arbitrary angle and ϕ' is the half of the angle subtended by the opening in the shell (Refer Figure 4-I), in Equation (4) the meridional force (N_ϕ) is obtained as

$$N_\phi = -\frac{ap}{2}\left(1 - \frac{\sin^2 \phi_0}{\sin^2 \phi}\right) \quad ...(6)$$

And hoop force (N_θ) can be evaluated from Equations (3) and (6) and expressed as

$$N_\theta = \frac{ap}{2}\left(1 - \frac{\sin^2 \phi_0}{\sin^2 \phi} - 2 \cos^2 \phi\right) \quad ..(7)$$

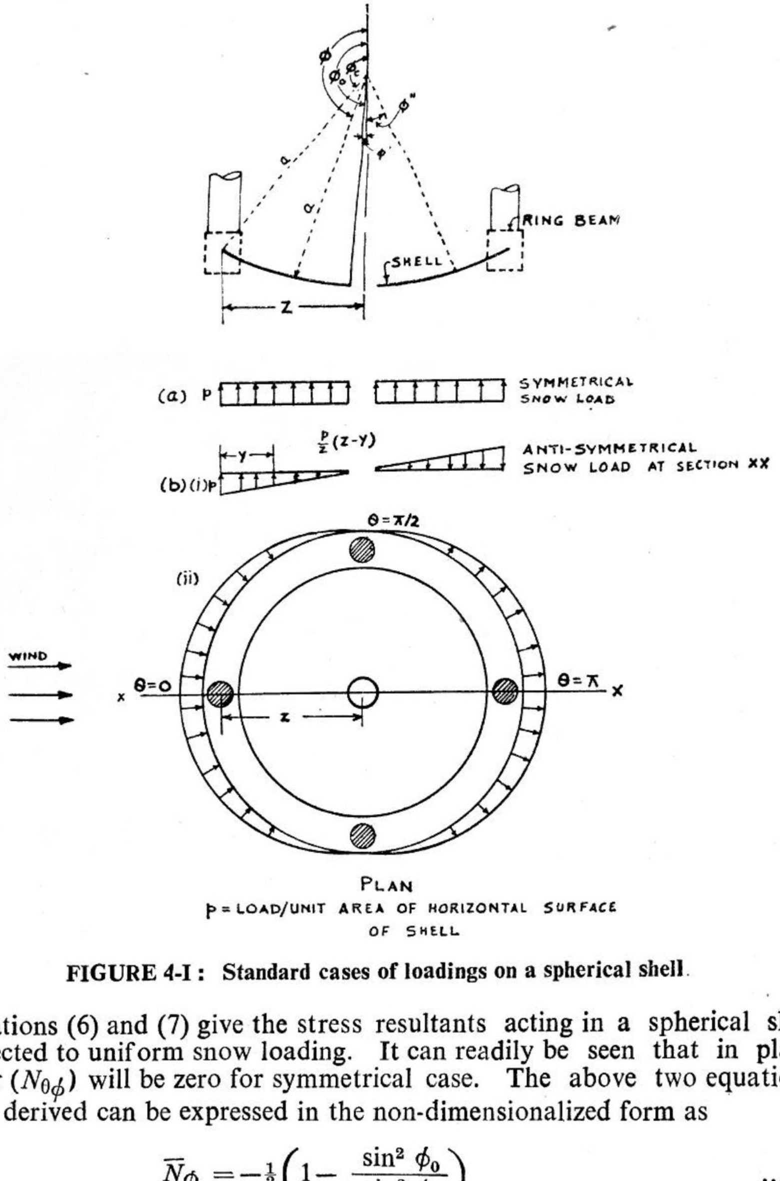


FIGURE 4-I : Standard cases of loadings on a spherical shell.

Equations (6) and (7) give the stress resultants acting in a spherical shell subjected to uniform snow loading. It can readily be seen that in plane shear ($N_{\theta\phi}$) will be zero for symmetrical case. The above two equations thus derived can be expressed in the non-dimensionalized form as

$$\overline{N}_{\phi} = -\tfrac{1}{2}\left(1 - \frac{\sin^2 \phi_0}{\sin^2 \phi}\right) \quad ...(8)$$

and

$$\overline{N}_{\theta} = \tfrac{1}{2}\left(1 - \frac{\sin^2 \phi_0}{\sin^2 \phi} - 2\cos^2 \phi\right) \quad ...(9)$$

where

$$\overline{N}_{\phi} = \frac{N_{\phi}}{ap} \text{ and } \overline{N}_{\theta} = \frac{N_{\theta}}{ap}$$

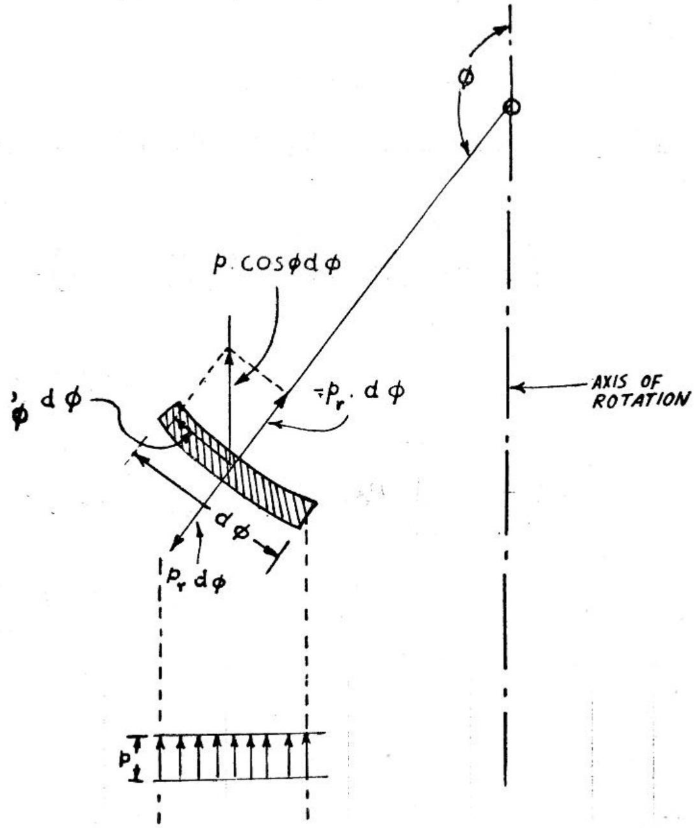


FIGURE 4-II : Loading components of the shell of revolution under snow load.

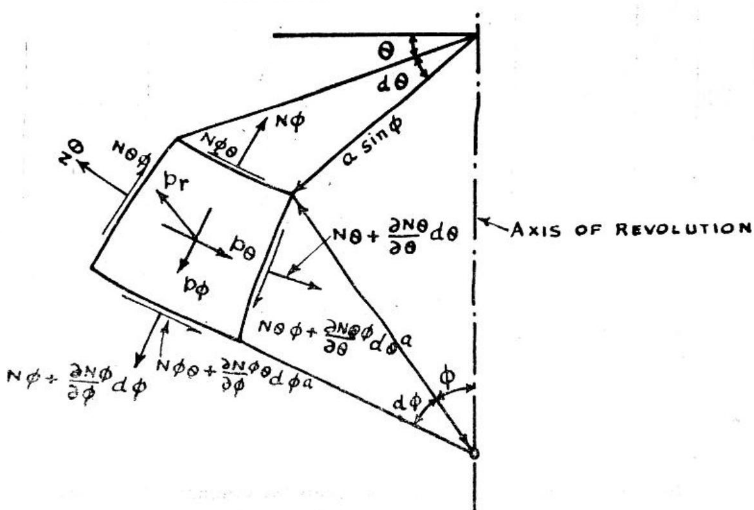


FIGURE 5 : Shell element.

Non-dimensionalized graphs based on Equations (8) and (9) are given in Figure 6.

(B) ANTI-SYMMETRICAL CASE

For anti-symmetric loading following explicit solutions are given by Flügge(2)

$$U_n = \frac{\cot^n \phi/2}{\sin^2 \phi}\left[A_n - a\int_{\phi'}^{\phi''}\left(p_{\phi n} + p_{\theta n} - \frac{n + \cos\phi}{\sin\phi}\, p_{rn}\right) \cdot \sin^2\phi \tan^n \phi/2 \, d\phi \right] \quad ...(10)$$

$$V_n = \frac{\tan^n \phi/2}{\sin^2 \phi}\left[B_n - a\int_{\phi'}^{\phi''}\left(p_{\phi n} - p_{\theta n} + \frac{n - \cos\phi}{\sin\phi}\, p_{rn}\right) \cdot \sin^2\phi \cot^n \phi/2 \, d\phi \right] \quad ...(11)$$

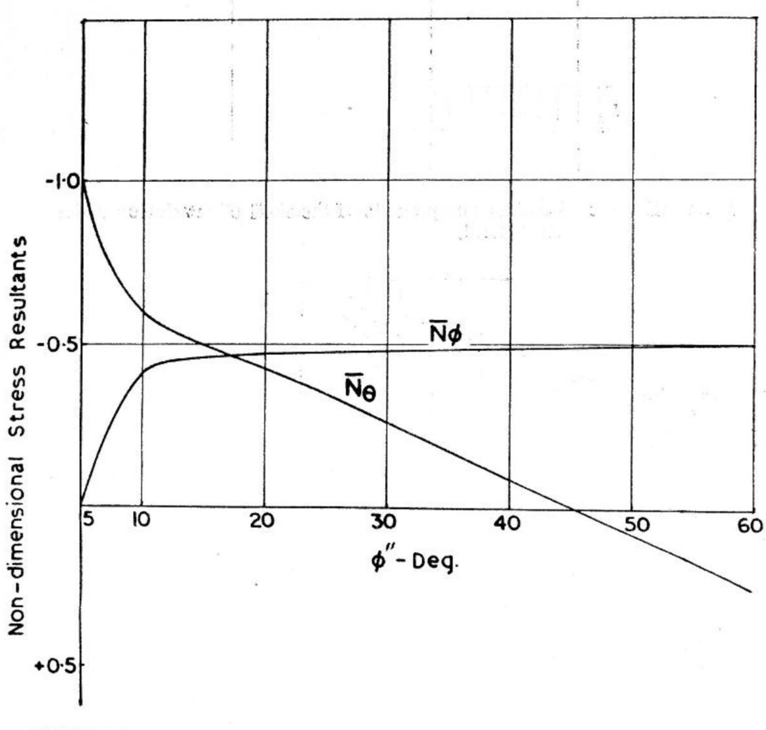


FIGURE 6 : Dimensionless stress resultants for symmetrical loading [$\phi'=5°$; $\overline{N}=N/ap$; +Tension ; —Compression].

where
$$N_\phi = \sum_0^n N_{\phi n} \cdot \cos n\theta$$
$$N_\theta = \sum_0^n N_{\theta n} \cdot \cos n\theta$$
$$N_{\theta\phi} = N_{\phi\theta} = \sum_1^n N_{\phi\theta n} \cdot \sin n\theta$$
$$N_{\theta n} = ap_{rn} - N_{\phi n}$$
$$U_n = N_{\phi n} + N_{\theta\phi n}$$
$$V_n = N_{\phi n} - N_{\theta\phi n} \qquad \text{...(12)}$$
$$\therefore \quad N_{\phi n} = \frac{U_n + V_n}{2}$$
and
$$N_{\theta\phi n} = \frac{U_n - V_n}{2}$$

Assuming only the first harmonic ($n=1$) the anti-symmetrical loading acting over the shell foundation is as shown in Figure 4-I. It is customary to assume $n=1$ for anti-symmetrical case and neglect higher harmonics because their contribution is not appreciable and on the other hand they make the analysis complicated.

Referring to Figure 4-I(b) the maximum stress distribution takes place along the section $x-x$ passing through the windward and leeward columns. Hence considering this worst case of loading, the ordinate of the loading at the section the distance Y from the windward column is equal to $p/Z\,(Z-Y)$ where Z is the radius of the plan of shell foundation.

Thus from the above discussions it follows that
$$p_{\phi_1} = \frac{p\,\phi(Z-Y)}{Z}$$
$$= \frac{p \sin^2 \phi}{\sin \phi_c} \cdot \cos \phi$$
Similarly,
$$p_{r1} = -\frac{p \sin \phi}{\sin \phi_c} \cdot \cos^2 \phi \qquad \text{...(13)}$$

Now applying limits $\phi=\phi'$ to $\phi=\phi''$ as discussed for symmetrical case in Equations (10) and (11) and substituting for loading components the expressions for U_1 and V_1 are
$$U_1 = \frac{\cot \phi/2}{\sin^2 \phi}\left[A_1 - \frac{ap}{\sin \phi_c} \; \frac{\sin^4 \phi - \sin^4 \phi_0}{4}\right] \qquad \text{...(14)}$$
$$V_1 = \frac{\tan \phi/2}{\sin^2 \phi}\left[B_1 - \frac{ap}{\sin \phi_c} \; \frac{\sin^4 \phi - \sin^4 \phi_0}{4}\right] \qquad \text{...(15)}$$

The two arbitrary constants A_1 and B_1 can be determined by applying the boundary conditions at

$$\phi=\phi_0,\ N_\phi=0 \text{ and } N_{\theta\phi}=0,\ i.e.,\ U_1=0 \text{ and } V_1=0$$

Hence it follows that $A_1=0$ and $B_1=0$. Now from Equations (12), (14), (15) and putting $n=1$ the values of N_ϕ, N_θ and $N_{\theta\phi}$ can be expressed as

$$N_\phi=-\frac{ap}{\sin\phi_c}\cdot\frac{\cos\theta}{\sin^3\phi}\left[\frac{\sin^4\phi-\sin^4\phi_0}{4}\right] \quad \text{...(16)}$$

$$N_{\theta\phi}=-\frac{ap}{\sin\phi_c}\cdot\frac{\cos\phi}{\sin^3\phi}\cdot\sin\theta\left[\frac{\sin^4\phi-\sin^4\phi_0}{4}\right] \quad \text{...(17)}$$

$$N_\theta=\frac{ap}{\sin\phi_c}\ \frac{\cos\theta}{4}\left[\frac{\sin^4\phi-\sin^4\phi_0}{\sin^3\phi}-4\sin\phi\ \cos^2\phi\right] \quad \text{...(18)}$$

The above equations can now be put into non-dimensionalized form by multiplying them by a factor $\sin\phi_c/ap$ on both sides. Thus the above equations reduce to

$$N^*_\phi=-\frac{1}{\sin^3\phi}\left[\frac{\sin^4\phi-\sin^4\phi_0}{4}\right]\cdot\cos\theta \quad \text{...(19)}$$

$$N^*_\theta=\frac{1}{4}\left[\frac{\sin^4\phi-\sin^4\phi_0}{\sin^3\phi}-4\sin\phi\ .\cos^2\phi\right].\ \cos\theta \quad \text{...(20)}$$

$$N^*_{\theta\phi}=-\frac{\cos\phi}{\sin^3\phi}\left[\frac{\sin^4\phi-\sin^4\phi_0}{4}\right]\cdot\sin\theta \quad \text{...(21)}$$

In order to have maximum membrane forces, θ must be zero; thus in Equations (19) and (20) $\cos\theta$ term vanishes and from Equation (21) $N^*_{\theta\phi}=0$. Non-dimensional graphs based on Equations (19) and (20) are given in Figure 7.

Domical shell is classified as a shell of revolution thus the surface for such a shell can be obtained by rotating a shutter about a central axis. Hence a rod is to be provided as a vertical axis in the centre of foundation. To accommodate this rod an opening is to be provided in the shell at the centre. Therefore, for all practical purposes the angles the opening subtends at centre of curvature may be taken as 10°. Hence while obtaining graphs for various stress resultants ϕ_0 is taken as 175°.

To illustrate the use of graphs in the design of domical shell foundation an example is taken up.

EXAMPLE

A domical shell foundation has 10 m. diameter. The position of the columns of the staging is as given in Figure 4-I. The maximum and minimum soil pressures under it are 4000 kg./m². and 2000 kg./m². respectively. Design the shell foundation.

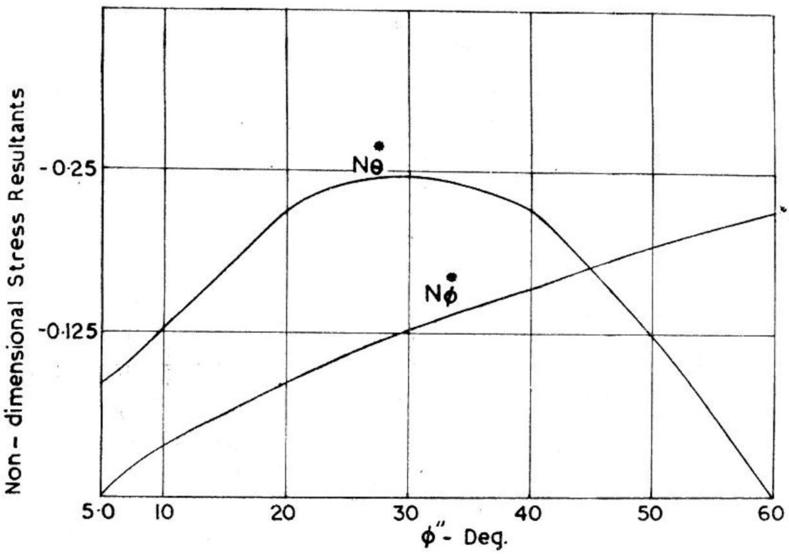


FIGURE 7 : Dimensionless stress resultants for anti-symmetrical loading

$\left[\theta=0\ ;\ \phi'=5°\ ;\ N^*=\dfrac{N \sin \phi_c}{ap};\ +\text{Tension}\ ;\ -\text{Compression}\right].$

SOLUTION

The diameter of the shell is given as 10 m. First of all it is essential to fix the radius 'a' and angle ϕ_c. To achieve this, two equations are to be formulated. Referring to Figure 8 first equation can be written as :

$$a \sin \phi_c = 500. \qquad ...(i)$$

The second equation is obtained by assuming the shell to be either deep or shallow. Vlasov(3) has suggested that a shell having $h/D<\frac{1}{5}$ (where h represents the maximum rise of the shell and D is the diameter of the shell in plan) can be treated as a shallow shell. To avoid constructional difficulties it is preferable to adopt shallow shell only, *i.e.*, keeping $h/D<\frac{1}{5}$, *then (Figure 4-I)*

$$\frac{1-\cos \phi_c}{\sin \phi_c} < 0{\cdot}4 \qquad ...(ii)$$

From Equations (i) and (ii) the shell parameters, *viz*, a and ϕ_c are to be fixed. Hence considering the above equations values of a and ϕ_c can be taken as, $a=777{\cdot}8$ cm. and $\phi_c=140°$.

Again the variable pressure as given in the problem can be split up into two parts ; one uniform load of 3000 kg/m². and other linearly varying load of 1000 kg./m². as shown in Figure 8.

Now from Figure 6 for symmetrical load corresponding to $\phi''=40°$ ($\phi_c=140°$) maximum dimensionless stress resultants are :

$$\overline{N}_{\phi,\ \max} = -0{\cdot}4908 \text{ (at } \phi''=40°)$$

$$\overline{N}_{\theta,\ \max} = -0.9926 \text{ (at } \phi''=5°)$$

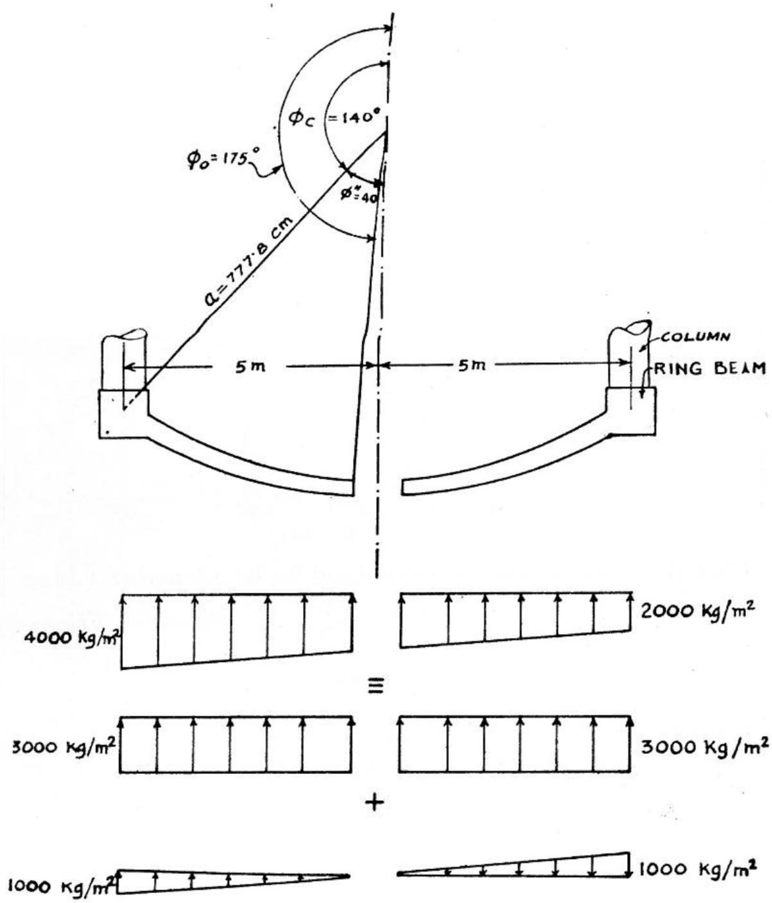


FIGURE 8 : Domical shell referred to in the numerical example.

The negative sign indicates that the shell is in compression. The numerical value of the maximum stress resultants can readily be calculated

$$^{S}N_{\phi,\ \max} = \overline{N}_{\phi,\ \max} \times ap$$

$$= -0{\cdot}4908 \times 777{\cdot}8 \times 0{\cdot}3 = -114{\cdot}5 \text{ kg./cm.}$$

$$^{S}N_{\theta,\ \max} = \overline{N}_{\theta,\ \max} \times ap$$

$$= -0{\cdot}9926 \times 777{\cdot}8 \times 0{\cdot}3 = -231{\cdot}6 \text{ kg./cm.}$$

Again from Figure 7 for anti-symmetrical loading for $\phi''=40$ ($\phi_c=140°$) maximum dimensionless stress resultants are :

$$N^{*}_{\phi,\ \max} = -0{\cdot}158 \text{ (at } \phi''=40°)$$

$$N^{*}_{\theta,\ \max} = -0{\cdot}245 \text{ (at } \phi''=30°)$$

Now the numerical value of the maximum stress resultants can be calculated by multiplying the dimensionless stress resultants by a factor $\frac{ap}{\sin \phi_c}$

$$^{A}N_{\phi,\,\max} = N^{*}_{\phi,\,\max} \times \frac{ap}{\sin\ \phi_c}$$

$$= -19{\cdot}12 \text{ kg./cm.}$$

$$^{A}N_{\theta,\,\max} = N^{*}_{\theta,\,\max} \times \frac{ap}{\sin\ \phi_c}$$

$$= -29{\cdot}65 \text{ kg./cm.}$$

Thus superimposing both the cases of loading the maximum stress resultants are obtained for trapezoidal type of loading (combined loading) :

$$^{C}N_{\phi,\,\max} = {}^{S}N_{\phi,\,\max} + {}^{A}N_{\phi,\,\max}$$

$$= 133.62 \text{ kg./cm. (compressive)}$$

$$^{C}N_{\theta,\,\max} = {}^{S}N_{\theta,\,\max} + {}^{A}N_{\theta,\,\max}$$

$$= 261.25 \text{ kg./cm. (compressive).}$$

Design of Shell

Assuming thickness of the shell = 10 cm.

Unit compressive stress = 261.25/10

= $26{\cdot}13$ kg./cm².

Taking strength of concrete M200 the allowable stress $= 2{\cdot}4 \times 70 = 28$ kg./cm². Hence the shell is safe in compression. As the whole shell is under compression, the nominal reinforcement is to be provided.

Hence area of steel to be provided $= 0{\cdot}3 \times 100 \times 10/100$

$= 3{\cdot}00$ sq. cm./m.

Provide 6 mm. diameter bars as hoops horizontally spaced at 9 cm. centre to centre. The same reinforcement may be provided along the meridian.

Design of Ring Beam

The ring beam is monolithic with shell as well as columns which support the superstructure. The ring beam is to be designed for uniform load which is coming from the bottom due to soil reaction. Thus it can be considered to be an inverted ring beam. With usual convention of negative support moment and positive mid-span moment its design can be carried out.

Conclusions

From Figures 6 and 7, the maximum values of stress resultants can readily be calculated for a domical shell of any size subjected to trapezoidal type of loading conventionally adopted for designing the foundations for tower-shaped structures. Once the values of the stress resultants are obtained the design of such foundations is extremely simple as can be seen from the example. Referring to the numerical example it is found that the meridional stresses caused due to anti-symmetrical loading is about 16 per cent of the stresses due to symmetrical loading. And in the case of hoop stress this percentage is only 13 per cent. Therefore, it is found that the contribution due to anti-symmetrical loading can very well be taken into account by increasing the stresses obtained due to symmetrical loading by 33⅓ per cent as suggested in I S. Code 456-1964. Consequently, the analysis due to anti-symmetrical loading may completely be avoided.

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