Behaviour of a Vertical Pile under Oblique Load

by

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Notations

The following symbols are used in this paper :

a = computed or assumed horizontal deflection of the pile at ground level,

b = computed or assumed slope of the pile at ground level,

c = second derivative of y with respect to x at ground level,

d = third derivative of y with respect to x at ground level,

EI = flexural rigidity of the pile,

e = base of Napierian logarithms,

H = elevation of point of application of load above ground level,

K_h = coefficient of horizontal subgrade reaction,

L = embedded length of pile,

M = moment at depth x,

M_g = moment at ground level,

n_h = constant of modulus of subgrade reaction,

P = axial load at the top of pile,

p = unit subgrade reaction,

Q = shear force normal to the undeflected axis of the pile,

Q_{hg} = shear load at ground level,

Q_t = horizontal load at the top of the pile,

W = inclined load acting on the pile,

x = depth below ground level,

y = displacement of the pile at depth x,

y_g = observed displacement of pile at ground level,

y_{tc} = computed or assumed displacement at the top of the pile,

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This paper was received on 29 June 1970. It is open for discussion up to June 1971.

α = ratio of vertical load to flexural rigidity (P/EI),

β = ratio of constant of modulus of subgrade reaction to the flexural rigidity (n_h/EI), and

θ = angle of inclination of load with horizontal.

Introduction

PILES and pile groups carrying vertical loads are frequently subjected to high lateral forces. These lateral forces may be either man made as in the case of lateral earth pressure, pile anchorages, pile supporting off-shore structures, *etc.*, or due to natural causes such as earthquakes, or wind forces. The deformation characteristics of piles and pile groups subjected to pure lateral loads have been studied by several investigators ([2, 3, 4, 8, 10, 11]). However, very limited data is available on the deflection of piles subjected to oblique loads (loads which have an axial and a lateral component) which is usually the case in practice.

Analytical solutions for the deflection of a pile subjected to purely lateral loads have been developed based on elastic theory which assumes that the soil behaves as a series of separate elements such as elastic springs. It is believed that this assumption can be applied to the problem of laterally loaded pile without a large error being caused([1, 14]). The authors believe that the same may be extended to the problem of a vertical pile subjected to oblique load. As shown in parts, "*a, b*" and "*c*" of Figure 1, ordinary beam theory can be used to develop the differential equation for a vertical pile subjected to oblique loads. The solution of the differential equation depends on the formulation of a mathematically convenient function for the soil reaction "*p*". As has been recognised, the soil reaction is a function of the pile properties, the stress strain relationships of the soil, effective unit weight of the soil, deflection of the pile, rate and inclination of loading and other parameters. Thus a complete solution to the problem of a vertical pile subjected to oblique loads has two major parts : first, it is necessary to obtain complete information describing the behaviour of the soil ; second, it is necessary to solve the differential equation. Between these two parts the first is by far the more difficult.

In this study the solution to the differential equation is obtained based on certain assumptions in respect of the soil behaviour. The values of deflection and slope (in respect of a single pile) are computed from the solution and compared with those measured from tests on a model pile. The following assumptions are made in respect of the soil reaction "*p*" :—

(1) If the soil consists of stiff clay, the coefficient of horizontal subgrade reaction has the same value "k_h" for every point of the surface of contact.

(2) For cohesionless soils, the value of the coefficient of horizontal subgrade reaction is represented by the relation

$$k_h = n_h \cdot x \quad ..(1)$$

The errors involved in these simplifying assumptions have been discussed in detail by Terzaghi([13]) and are believed to be within permissible limits. It may be mentioned here that analysis of test data (7) has shown that the solution based on assumption made in respect of stiff clays, provides the best possible correlation between the field test data and calculated values in respect of piles driven in soft clays, too. However, more field test data and experimental data are desirable to extend the validity of this assumption for soft clays.

In practice, piles or pile groups are partially restrained at the top and do always carry axial loads when subjected to lateral forces. With a view to simulate this field condition, the authors are prompted to take up the analytical and experimental study of deflections of a free headed and partially embedded single pile subjected to oblique loads. The deflections recorded in practice, because of usual restraint at top, will be less than those observed in these tests. However, a free headed pile is adopted for the reason that, in model testing with relatively small loads, large deflections are obtained facilitating accurate measurement. Also test data available indicate that pile groups behave similarly as single piles at working loads and that total resistance can be calculated as the sum of the resistances of individual piles under identical conditions(6).

Analytical Solutions

The solution of the governing differential equation for the deflection of a vertical pile subjected to inclined load is obtained for two particular cases : (*i*) when the coefficient of horizontal subgrade reaction is an absolute constant and (*ii*) when it is a linear function of depth. The solution in the first case is obtained in a closed form involving exponential and trigonometric functions and the resulting elastic curve is a damped sine wave. In the second case when the coefficient of horizontal subgrade reaction varies linearly with depth, a series solution is obtained.

Figure 1 shows the partially embedded vertical pile subjected to an oblique load. The governing differential equation for the problem is (5, 6)

$$EI\frac{d^4y}{dx^4}+p\,\frac{d^2y}{dx^2}=0 \qquad ...(2)$$

for the portion of the pile above ground surface as there is no soil reaction. The solution for this equation is available for all cases of loading.

For the embedded portion of the pile, the differential equation is

$$EI\,\frac{d^4y}{dx^4}+p\,\frac{d^2y}{dx^2}+k_h.y=0 \qquad ...(3)$$

The boundary conditions for the problem are as follows :

At the ground level,

Applied Moment $M=EI\,y''(0)=E\,.\,I\,.\,c.$ ...(4)

where $c=y''(0)$

and applied shear $Q_{ho}=EIy'''(0)=EId.$ ...(5)

where $y'''(0)=d$

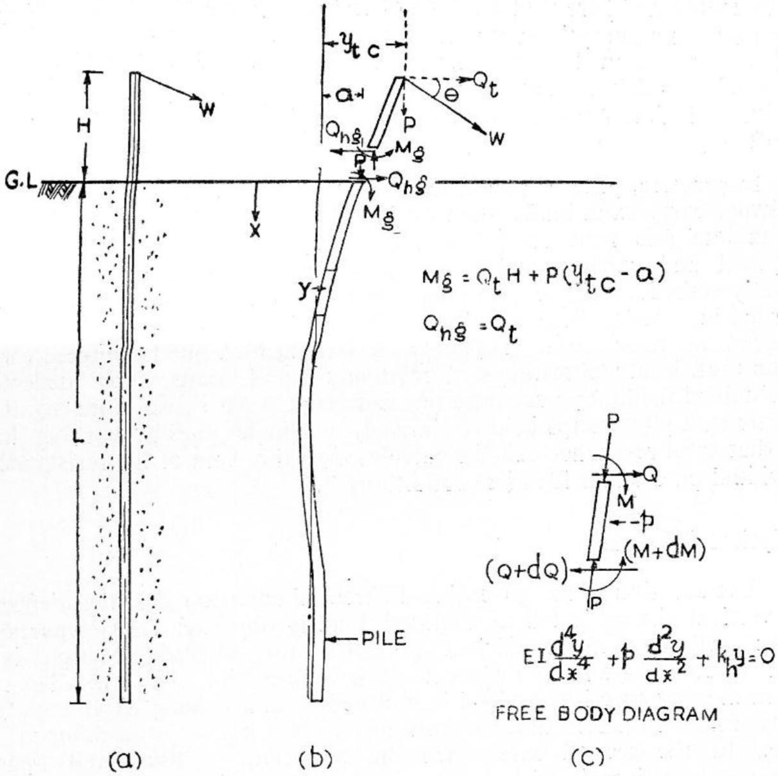

FIGURE 1 : The problem of a partially embedded single vertical pile subjected to oblique load.

At the lower end of the pile, both the moment and shear are zero, *i.e.*,

$$y''(L)=0 \quad \text{...(6)}$$

and

$$y'''(L)=0 \quad \text{...(7)}$$

In Equations (4) and (5) the values of "*c*" and "*d*" are known characterising the applied moment and applied shear at the ground level. When k_h is an absolute constant in Equation (2) which is applicable for stiff clays, the solution for the elastic line is obtained in a closed form involving trigonometric and exponential functions :

$$y = e^{(-x R \cos \phi)} [A \cos (xR \sin \phi) + B \sin (xR \sin \phi)] \quad \text{...(8)}$$

where

$$R = \sqrt[4]{\frac{p^2+\gamma^2}{4 E^2 I^2}}$$

$$\phi = (-\tfrac{1}{2}) \tan^{-1} (\gamma/p)$$

and

$$-\gamma^2 = (p^2 - 4 k_h EI)$$

The constants A and B in Equation (8) are determined from the boundary conditions [Equation (4) through Equation (7)]. When k_h varies linearly with depth, *i.e.*, $k_h = n_h\, x$, the Equation (3) can be written as

$$\frac{d^4 y}{dx^4} + \alpha \frac{d^2 y}{dx^2} + \beta x\, y = 0 \qquad \ldots(9)$$

where

$$\alpha = \frac{P}{EI} \text{ and } \beta = \frac{n_h}{EI}$$

The solution for this equation is obtained in the series form

$$y = x^r \sum_{n=0}^{n=\infty} C_n\, x^n \qquad \ldots(10)$$

where $C_o \neq 0$ and subject to boundary conditions [Equation (4) through Equation (7)]

$$\text{let } y\,(0) = a \qquad \ldots(11)$$

$$\text{and } y'\,(0) = b \qquad \ldots(12)$$

be the deflection and slope at ground level. The computation of "a" and "b" is facilitated by the following procedure. First a solution of Equation (10) subject to the boundary conditions [Equations (4), (5), (11) and (12)] is obtained. When the boundary conditions as per Equations (6) and (7) are substituted in the solution, two simultaneous equations are obtained, from which the values of a and b are evaluated. The two simultaneous equations thus obtained, retaining terms up to the third order in α and β, are :

$$a\left[\beta\left(\frac{-L^3}{3!} + \frac{L^5}{5!}\alpha - \frac{L^7}{7!}\alpha^2\right) + \beta^2\left(\frac{6L^8}{8!} - \frac{14L^{10}}{10!}\alpha\right) + \beta^3\left(\frac{-66L^{13}}{13!}\right)\right]$$

$$+\, b\left[2\beta\left(\frac{-L^4}{4!} + \frac{L^6}{6!}\alpha - \frac{L^8}{8!}\alpha^2\right) + 2\beta^2\left(\frac{7L^9}{9!} - \frac{16L^{11}}{11!}\alpha\right) + 2\beta^3\left(\frac{-84L^{14}}{14!}\right)\right]$$

$$+\, c\left[\left(1 - \frac{L^2}{2!}\alpha + \frac{L^4}{4!}\alpha^2 - \frac{L^6}{6!}\alpha^3\right) + \beta\left(\frac{-3L^5}{5!} + \frac{8L^7}{7!}\alpha - \frac{15L^9}{9!}\alpha^2\right) + \beta^2\left(\frac{24L^{10}}{10!} - \frac{104L^{12}}{12!}\alpha\right) + \beta^3\left(\frac{-312L^{15}}{15!}\right)\right]$$

$$+\, d\left[\left(L - \frac{L^3}{3!}\alpha + \frac{L^5}{5!}\alpha^2 - \frac{L^7}{7!}\alpha^3\right) + \beta\left(\frac{-4L^6}{6!} - \frac{10L^8}{8!}\alpha - \frac{18L^{10}}{10!}\alpha^2\right) + \beta^2\left(\frac{36L^{11}}{11!} - \frac{146L^{13}}{13!}\alpha\right) + \beta^3\left(\frac{-504L^{16}}{16!}\right)\right] = 0 \qquad \ldots(13)$$

and

$$a\left[\beta\left(\frac{-L^2}{2!}+\frac{L^4}{4!}\alpha-\frac{L^6}{6!}\alpha^2\right)+\beta^2\left(\frac{6L^7}{7!}-\frac{14L^9}{9!}\alpha\right)+\beta^3\left(\frac{-66L^{12}}{12!}\right)\right]$$

$$+b\left[2\beta\left(\frac{-L^3}{3!}+\frac{L^5}{5!}\alpha-\frac{L^7}{7!}\alpha^2\right)+2\beta^2\left(\frac{7L^8}{8!}-\frac{16L^{10}}{10!}\alpha\right)+2\beta^3\left(\frac{-84L^{13}}{13!}\right)\right]$$

$$+c\left[\left(\frac{-L}{1!}\alpha+\frac{L^3}{3!}\alpha^2-\frac{L^5}{5!}\alpha^3\right)+\beta\left(\frac{-3L^4}{4!}+\frac{8L^6}{6!}\alpha-\frac{15L^8}{8!}\alpha^2\right)+\beta^2\left(\frac{24L^9}{9!}-\frac{104L^{11}}{11!}\alpha\right)+\beta^3\left(\frac{-312L^{14}}{14!}\right)\right]$$

$$+d\left[\left(1-\frac{L^2}{2!}\alpha+\frac{L^4}{4!}\alpha^2-\frac{L^6}{6!}\alpha^3\right)+\beta\left(\frac{-4L^5}{5!}+\frac{10L^7}{7!}\alpha-\frac{18L^9}{9!}\alpha^2\right)+\beta^2\left(\frac{36L^{10}}{10!}-\frac{146L^{12}}{12!}\alpha\right)+\beta^3\left(\frac{-504L^{15}}{15!}\right)\right]=0 \quad \ldots(14)$$

The values of 'a' and 'b' thus obtained are compared with the experimental results.

Properties of Materials Used

(A) MODEL PILES

The model piles used in the tests were aluminium pipes supplied by Indian Aluminium Company, Calcutta having the following properties :—

(*i*) Section number : INDAL 9442

(*ii*) Outer diameter : D : 1·5 cm.

(*iii*) Inner diameter : d : 1·0 cm.

(*iv*) Modulus of elasticity : E : $0{\cdot}724\times10^6$ kg./sq. cm.

(*v*) Moment of inertia : I : 180×10^{-3} cm^4.

(*vi*) Flexural rigidity : EI : 13×10^4 kg. cm^2.

(B) SOIL

The properties of the uniform dry sand used in the investigation are :

(*i*) Soil type : Dry medium and uniform sand

(*ii*) Effective size D_{10} : 0·25 mm.

(*iii*) Uniformity coefficient : 1·44

(*iv*) Specific gravity : 2·63

(*v*) Maximum void ratio : 0·95

(*vi*) Minimum void ratio : 0.60

For all tests conducted, the density of the sand compacted with piles in position is kept at a constant value of 1·49 gm./c.c. which gives a relative density of 54 per cent for the sand used.

Description of Experimental Set-up

In order to arrive at the optimum length of the model piles for the study, preliminary tests were conducted on short piles 20 cm. long and the constant of horizontal subgrade reaction n_h was computed. The results have indicated that the size of the pile chosen for the study, has to be embedded to a depth of 60 cm. so that it could be considered as flexible pile (*i.e.*, non-dimensional depth coefficient $Z_{max.} = L/T$ is greater than 5, where L = embedded length of the pile and T = relative stiffness factor defined by the equation $T = \sqrt[5]{\frac{EI}{n_h}}$). Hence 60 cm. length of embedment was adopted for all model pile tests conducted.

The experimental set-up is shown in Figure 2. The tank is built up of wooden planks stiffened with angle irons on all edges and its dimensions are 114 cm. × 60 cm. × 75 cm. It was considered optimum to keep four piles in position in a row each time the tank is filled up with the soil. The four piles were kept at a distance of 28 cm. centre to centre in the direction of the length of the tank giving a clearance of 15 cm. from the edge of the tank to the nearest pile. The edge clearance in the perpendicular direction was 30 cm. for each pile. The bottom of each pile was kept at

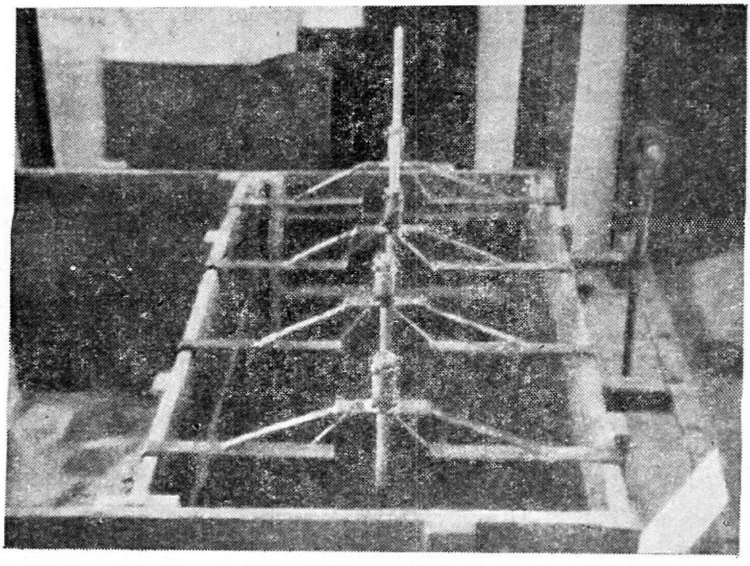

FIGURE 2 ; Tank assembly.

a height of 15 cm. above the base of the tank. The spacing of the piles and the edge clearances adopted are adequate to ensure that the behaviour of each pile under loading is unaffected by either the stressed zone of an adjacent pile where loaded, or the presence of the tank sides(9) (Figure 2).

Method of keeping the Piles in Position during the Operation of Filling the Tank with Sand

The following procedure is adopted for the purpose of fixing the piles in position while compacting the sand. A supporting frame is designed for this purpose. The frame has at its centre a steel tube 7·5 cm. long and whose external and internal diameters are 2·5 and 1·5 cm. respectively. The tube is provided with 8 screws distributed along its length and in different radial directions to grip firmly the pile passing through the tube. First, the supporting frame is fixed to the tank firmly by means of screws and the pile is passed through the tube and the screws are tightened so that the pile is gripped firmly in position. The verticality of the pile was checked by means of a suitable spirit level. After the tank is completely filled with sand, the screws gripping the pile are first loosened gently and then the screws, which fasten the supporting frame to the tank are loosened and then the frame is taken out gently with little disturbance to the pile leaving it in the required position.

Placing and Compacting the Sand

After keeping the piles in position, a measured quantity of sand was poured and spread uniformly to form a layer of 9·5 cm. thick and made level. The sand layer was compacted by means of a hammer weighing 2 kg. and with a free fall of 24 cm. To ensure uniform compaction the sliding weight of the hammer is allowed to fall on a square base attached to the central rod and resting on the sand. The number of blows for each layer was 55. This procedure was finally adopted only after preliminary tests have indicated good reproducibility of the desired density for the soil compacted.

Application of Load and Measurement of Deflection

Application of horizontal or oblique loads on the pile was done by means of a wire rope passing over the ball bearing pulley mounted on the side of the tank. The top end of the wire rope is formed into a loop and attached to the hook which is connected to a hinge made at the top of the pile. The hinge is formed by passing a small rod through a hole at the top of the pile. The lower end of the wire rope carries a load hanger over which the required loads are placed. The pulley on the side of the tank can be adjusted vertically to any desired height opposite to any one of the four piles in the tank. Loads were applied in increments of 0·909 kg. (2 lb.) for the first 9.09 kg. (20 lb.) and then in increment of 2·27 kg. (5 lb.) up to 27·27 kg. (60 lb.). For each increment of load, 5 minutes time was allowed to elapse before the deflections were measured and next increment of load applied (Figure 3).

For each load application on the pile, the horizontal deflections at the ground surface, and at the top of the pile, are measured, by means of dial-gauges mounted suitably on a stand with magnetic base which can be readily fixed on a steel plate mounted to the side of the tank. The shaft

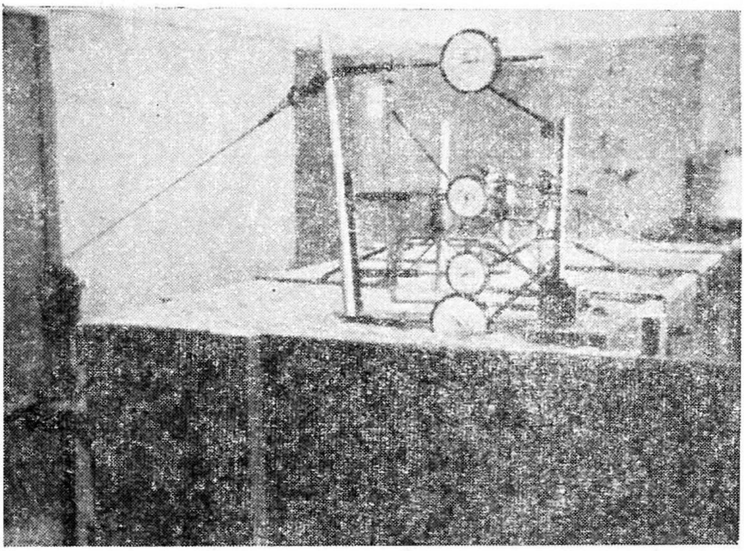

FIGURE 3 : Inclined load test.

of the dial-gauge is placed against a 2 cm. × 2 cm. flat surface connected by means of a sleeve through a hinge provided in the pile. The slope of the pile at the ground surface is found by measuring the increase in the horizontal deflection of a vertical shaft connected to the pile at the ground surface, over a length of 10 cm. above the ground level. All the tests were repeated twice to ensure the reproducibility.

Presentation and Analysis of Test Results

Figure 4 represents a graph drawn to show the relationship between the load applied horizontally to a rigid pile (whose top is flush with the ground) at the ground level at the horizontal deflection of the top of the pile in order to compute the constant of horizontal subgrade reaction, n_h.

The horizontal deflection of the top of the pile (projecting above the ground level) due to lateral/oblique load applied at the pile top is shown in Figure 5. Similar tests have been conducted on four different piles whose tops are 0, 10.16 cm. (4 in.), 20.32 cm. (8 in.) and 30.48 cm. (12 in.) respectively above the ground level. The actual directions of oblique loads applied are varied with each of the piles tested as shown therein (Figure 5).

Similar relationships between the horizontal deflection of the pile at ground level and the magnitudes of the oblique loads applied at the top of the pile are plotted in Figure 6. Figure 7 gives the relation between the load applied at the top of the pile and the slope of the pile at ground level.

An analysis of the curves presented in Figures 6 and 7 leads to the following inferences :—

(1) For a given magnitude of the oblique load applied at the pile top above ground level the horizontal deflection and slope of the

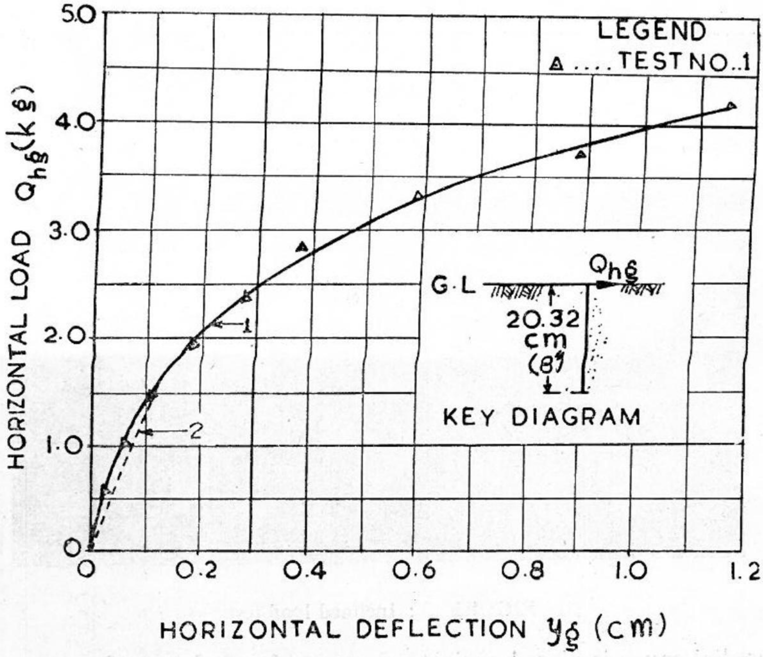

FIGURE 4 : Load versus deflection at ground level.

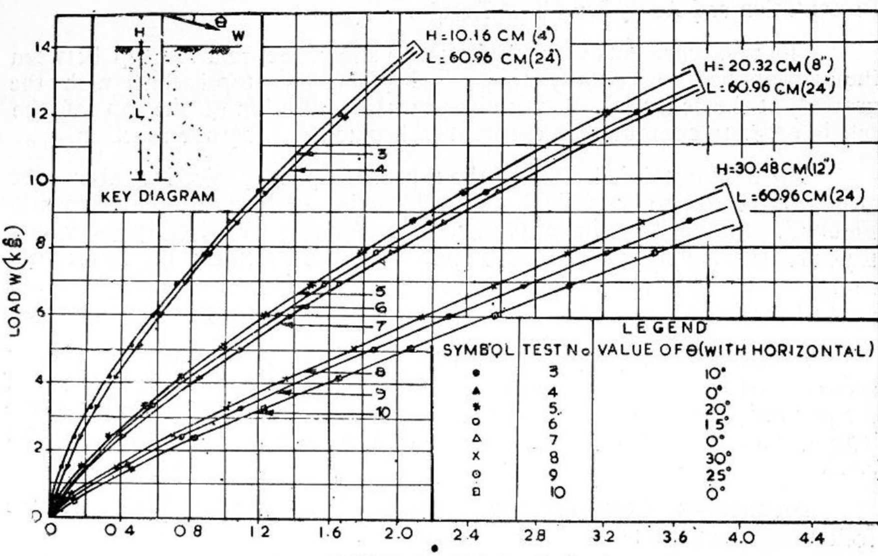

FIGURE 5 : Load versus horizontal deflection at top of pile.

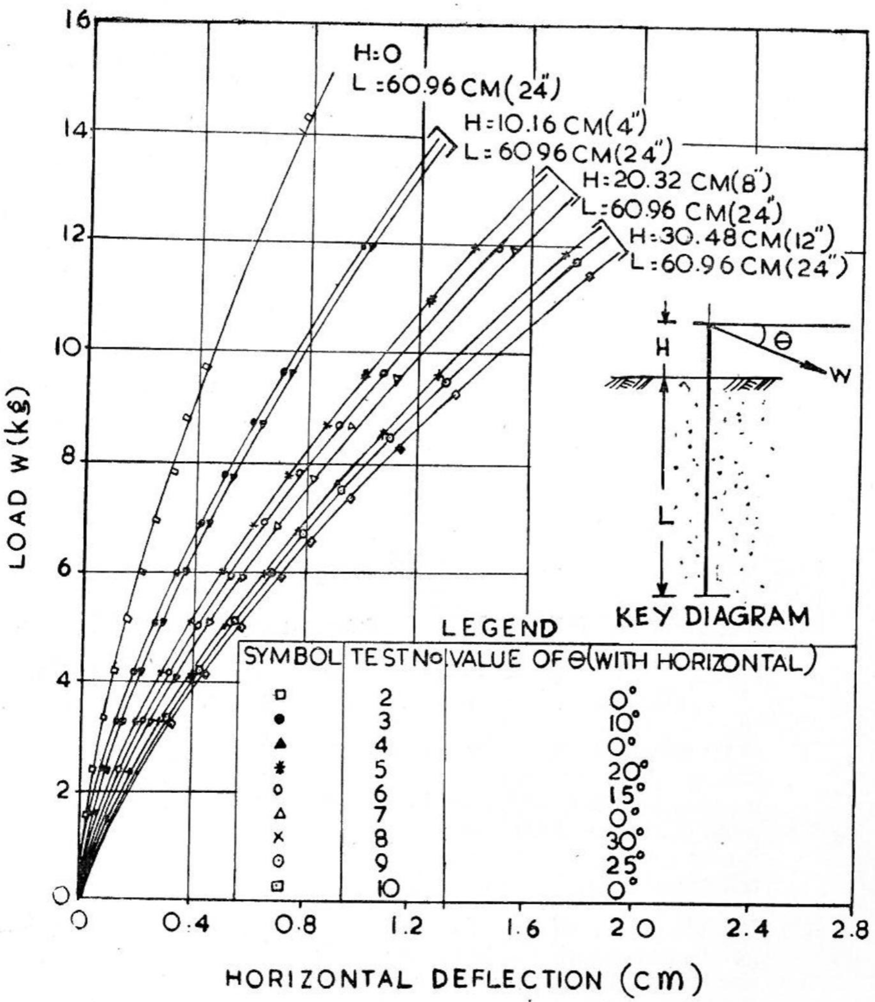

FIGURE 6 : Load versus horizontal deflection at ground level.

pile at ground level decrease with increase in the downward inclination of the oblique load. Thus the worst case occurs when a given load at the pile top acts horizontally. This can be explained as follows : The oblique load can be considered as a combination of its horizontal and vertical components. For a given magnitude of the oblique load, its horizontal component decreases with increase in its inclination (downwards with the horizontal). The applied moment and shear and hence the resulting slope and deflection of the pile at ground level are smaller due to an oblique load (inclined downwards) than those that would have been caused if the same load were applied horizontally. However, the top of the pile is shifted laterally by a small amount owing to the horizontal component of the applied load. This, in its turn causes a slight increase in the moment

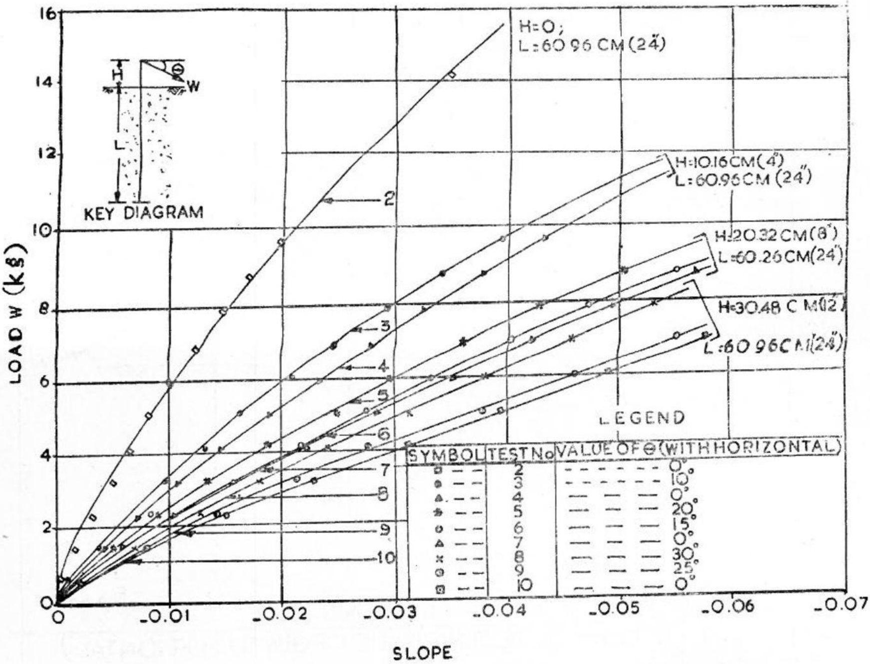

FIGURE 7 : Load versus slope at ground level.

at the ground level of the pile on account of the vertical component of the oblique load. This slight increase in moment tends to augment the slope and deflection of the pile at ground level. But this contribution is of a lower order. This inference is also confirmed by the results obtained by the authors from tests conducted (Test Nos. 3 to 10) in this study.

(2) It is also observed that for a given magnitude, the higher the point of application of horizontal/oblique load the greater are the horizontal deflection and slope of the pile at gound level. Evidently this is due to the increase in the moment applied to the pile at ground level.

Computation of Theoretical Load-Deflection Curves

For computing the load-deflection curves [based on the solution of the differential Equations (3) and (4)] the value of the constant of horizontal subgrade reaction n_h, is to be evaluated. This is done from both rigid and flexible pile test data as follows :

Approximating the initial one-third load-deflection curve to be linear for a rigid pile under horizontal load, the value of n_h is calculated from the relationship(13).

$$n_h = \frac{18\, Q_{hg}}{y_g\, L^2} \qquad ...(15)$$

and is found to be 0.626 kg./cm³. (Figure 2).

The values of deflection y_g, shear Q_{hg} and moment M_g at ground level as obtained from a flexible model pile test data are substituted in the following relation ([10])

$$y_g = \frac{A_y \, Q_{hg} \, T^3}{EI} + \frac{B_y \, . \, M_g \, T^2}{EI} \qquad \text{...(16)}$$

where $A_y = 2.38$ is the non-dimensional coefficient for shear ([10]).

$B_y = 1.63$ is the non dimensional coefficient for moment ([10]).

Knowing the value of flexural rigidity of the pile tested, the value of T is computed and from the relation $T = \sqrt[5]{\frac{EI}{n_h}}$, the value of n_h is evaluated. Using the values of n_h (obtained from both rigid and flexible pile test data) the deflection and slope at the ground level of a flexible pile under given oblique load are obtained utilizing the solution developed by the authors [Equations (13) and (14)] by adopting the method of successive approximations as detailed below :

Neglecting the moment at ground level due to the vertical component of the applied oblique load, the values of shear and moment at ground level are equal to Q_t and $Q_t.H$ respectively (Figure 1). Using these quantities, the values of "c" and "d" are calculated from Equations (4) and (5). From Equations (13) and (14) the values of "a" and "b" the deflection and slope at ground level respectively are computed. Also the deflection at the top of the pile is calculated. From these the values of deflection of the pile at its top and ground level, the moment at ground level is calculated to a second approximation as given by the relation

$$M_g = Q_t.H + P(y_{tc} - a) \qquad \text{...(17)}$$

The Equations (13) and (14) are again solved for "a" and "b" using the revised moment at ground level as obtained from Equation (17). It has been found that the moment at ground level contributed by the vertical component of the oblique load is of the order of 2 per cent of the moment contributed by the horizontal load.

It has been found previously([12]) that the results of theoretical computations based on the values of n_h from rigid pile tests were not in reasonable agreement with the observed values of deflections for flexible piles. For this reason, Equations (13) and (14) have been solved by the substitution of a constant value of

$$\beta = \frac{n_h}{EI} \text{ computed from rigid pile test data.}$$

Further the Equations (13) and (14) have been solved for three different values of β obtained from flexible pile test data (Test No. 3) in order to account for the variation of n_h with deflection.

Curve 1 in Figure 8 gives the relation between the observed deflection and the load in the case of flexible pile subjected to an oblique load (Test No. 3). In order to obtain the load-deflection curve by computation based on flexible pile test data, typical values of load (5·09 kg; 9·64 kg, and 14.20 kg.)

covering the full range of loading are taken and the corresponding deflections are computed using the Equations (13) and (14) developed by the authors. The curve thus obtained is marked No. 2 in the figure. As can be readily seen, there is good correlation between the observed and the computed deflections of the pile. Curve marked No. 3 in Figure 8 shows the computed load-deflection curve for the same pile in the same manner using the rigid pile test data. As seen from the curves, there is good correlation between the computed and observed deflections up to about 1/3rd the maximum load beyond which computed deflections are found to be increasingly lower than the observed deflections. This clearly indicates that n_h cannot be taken to be a constant.

Identically similar findings are obtained in respect of computed and observed values of slope at ground level (*vide* curves marked *1, 2 and 3* in Figure 9). It may be mentioned here that a more accurate shape for curve No. 2 in Figures 8 and 9 can be obtained by taking a greater number of computed points. In view of the time consuming computations involved, only three points were included in this study. However, a computer can be advantageously used to increase the accuracy of the curve.

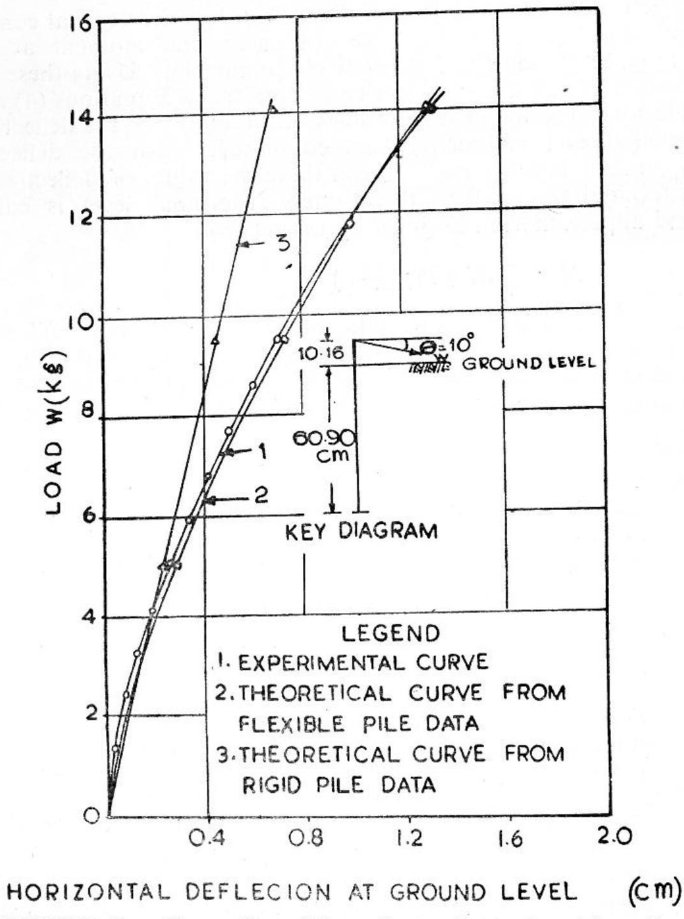

FIGURE 8 : Comparison of theoretical and experimental results.

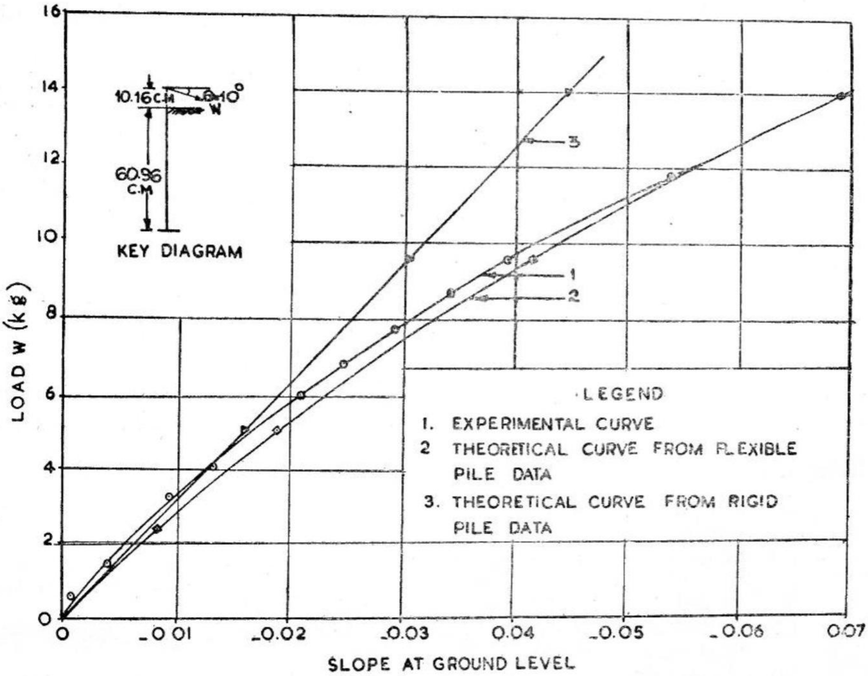

FIGURE 9 : Comparison of theoretical and experimental results.

The difference between the observed and computed values based on rigid pile test data in respect of both slope and deflection may be attributed to the following factors :

The values of n_h are assumed to be constant [as suggested by Terzaghi(13)] and proportional to the slope of the straight line marked 2 in Figure 4, while actually n_h varies and is proportional to the slope of the curve marked 1 in Figure 4. As such the authors are prompted to conclude that the variation in the value of n_h should be recognized and included for the computation of deflection of an obliquely loaded pile.

Conclusions

(1) The analytical solution of the basic differential equation governing the problem of a vertical pile subjected to inclined loads above ground level is developed in the series form. The theoretical values of deflection and slope at ground level calculated from the solution compare favourably with the observed values from tests on model piles.

(2) A purely horizontal load applied at the top of a pile produces greater deflection and slope at ground level than those produced by a downwardly inclined load of the same magnitude and acting at the same point.

(3) The horizontal deflection and slope at ground level decrease with increase in the downward inclination of the oblique loads acting at any given elevation above ground level.

(4) Due to a given horizontal load acting at the top of the pile, the more the distance between the top of the pile and ground level,

the larger are the horizontal deflection and slope of the pile at ground level.

(5) With the limited test data obtained, it is fair to conclude that the variation in the value of the constant of horizontal subgrade reaction, n_h varies with the load range and is not a constant as originally assumed by Terzaghi(13). Also the extension of and use of the non-dimensional factors given by Reese and Matlock(10) to piles subjected to oblique loads is indicated.

Acknowledgement

The authors express their gratitude to Group Captain M.J. Kirpalani, Principal, Regional Engineering College, Warangal-4, Andhra Pradesh, for his encouragement and to Dr. N.Ch. Pattabhiramacharyulu, of Mathematics Department, Regional Engineering College, Warangal-4, Andhra Pradesh, for his help in obtaining the solutions of the differential equations.

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